

Handling Infinitely Branching WSTS

Michael Blondin^{1 2}, Alain Finkel¹ & Pierre McKenzie^{1 2}

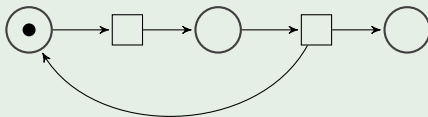
¹LSV, ENS Cachan

²DIRO, Université de Montréal

July 8, 2014

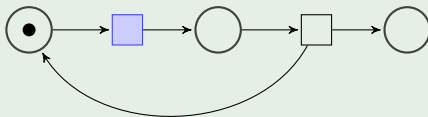
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Example of WSTS: Petri nets (Geeraerts, Heußner, Praveen & Raskin PN'13)



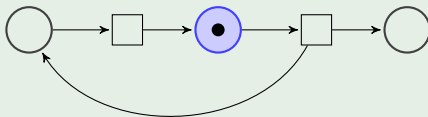
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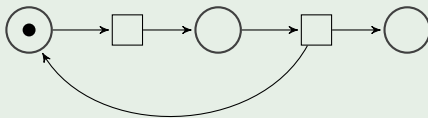
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Multiple decidability results are known for finitely branching WSTS.

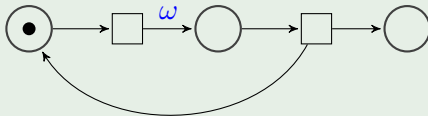
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$$\text{Post}(\odot \circ \circ) = \circ \odot \circ$$

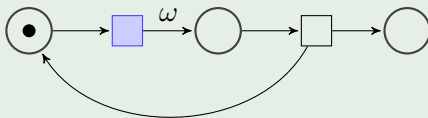
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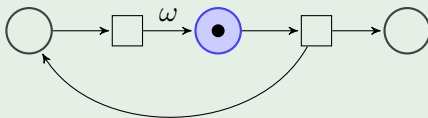
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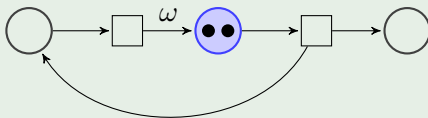
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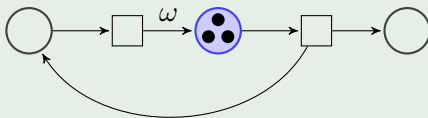
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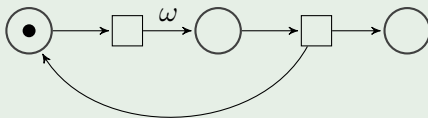
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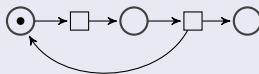


$$\text{Post}(\odot \circ \circ) = \circ \odot \circ, \circ \odot \odot \circ, \circ \odot \odot \odot \circ, \dots$$

Well-structured transition system (Finkel ICALP'87, Finkel & Schnoebelen TCS'01)

$S = (X, \rightarrow, \leq)$ where

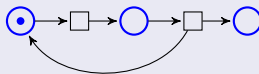
- X set,
- $\rightarrow \subseteq X \times X$,
- monotony,
- well-quasi-ordered.



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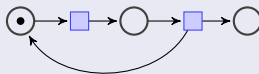
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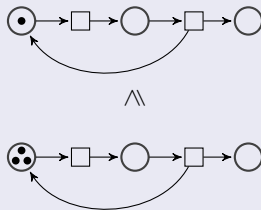
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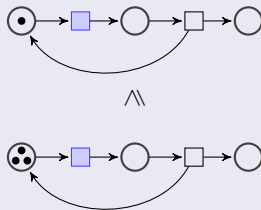
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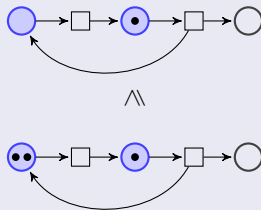
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$S = (X, \rightarrow, \leq)$ where

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- **transitive** monotony,
- well-quasi-ordered.

$$\forall x \rightarrow y$$

$$\wedge$$

$$x' \rightarrow y'$$

$$\wedge$$

$$x \leq x' \wedge y \leq y' \Rightarrow x' \rightarrow y'$$

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$S = (X, \rightarrow, \leq)$ where

- X set,
- $\rightarrow \subseteq X \times X$,
- monotony,
- well-quasi-ordered:
 $\forall x_0, x_1, \dots \exists i < j$ s.t. $x_i \leq x_j$.

Branching

A WSTS (X, \rightarrow, \leq) is *finitely branching* if $\text{Post}(x)$ is finite for every $x \in X$.

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- Much more.

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- Parametric WSTS.

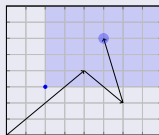
Objective

We want to study the usual reachability problems for these infinitely branching systems, e.g.,

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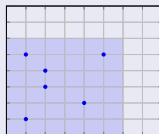
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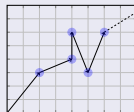
- Termination,
- Coverability,
- Boundedness.



Termination

Input: (X, \rightarrow, \leq) a WSTS, $x_0 \in X$.

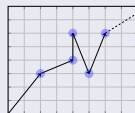
Question: $\exists x_0 \rightarrow x_1 \rightarrow x_2 \rightarrow \dots?$



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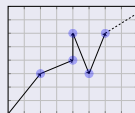
Theorem (Finkel ICALP'87)

Termination is decidable for finitely branching WSTS with transitive monotony.

Termination

Input: (X, \rightarrow, \leq) a WSTS, $x_0 \in X$.

Question: $\exists x_0 \rightarrow x_1 \rightarrow x_2 \rightarrow \dots?$



Theorem (deduced from Dufourd, Jančar & Schnoebelen ICALP'99)

Termination is undecidable for infinitely branching WSTS.

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Remark

Strong termination and termination are the same in finitely branching WSTS.

Strong termination

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Question: $\exists k$ bounding length of executions from x_0 ?

Theorem

Strong termination is decidable for infinitely branching WSTS under some assumptions.

Issues with finite branching techniques

Some techniques for WSTS based on finite reachability trees; impossible for infinite branching.

Some rely on upward closed sets; what about downward closed, in particular with infinite branching?

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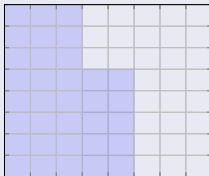
A tool

Develop from the WSTS *completion* introduced by Finkel & Goubault-Larrecq 2009.

Ideals

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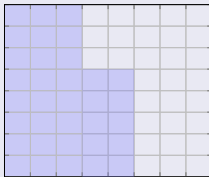
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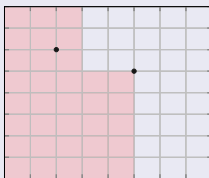
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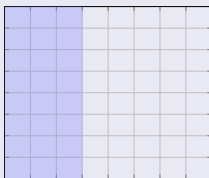
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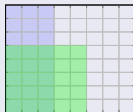
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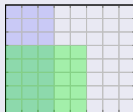
Theorem (Finkel & Goubault-Larrecq ICALP'09; Goubault-Larrecq '14)

D downward closed $\implies D = \bigcup_{\text{finite}} \text{Ideals}$



Theorem (Finkel & Goubault-Larrecq ICALP'09; Goubault-Larrecq '14)

$$D \text{ downward closed} \implies D = \bigcup_{\text{finite}} \text{Ideals}$$



Corollary

Every downward closed set decomposes canonically as the union of its maximal ideals.

Completion

The *completion* of $S = (X, \rightarrow_S, \leq)$ is $\widehat{S} = (\widehat{X}, \rightarrow_{\widehat{S}}, \subseteq)$ such that

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Completion

The *completion* of $S = (X, \rightarrow_S, \leq)$ is $\widehat{S} = (\widehat{X}, \rightarrow_{\widehat{S}}, \subseteq)$ such that

- $\widehat{X} = \text{Ideals}(X)$,
- $I \rightarrow_{\widehat{S}} J$ if $\downarrow \text{Post}(I) = \underbrace{\dots \cup J \cup \dots}_{\text{canonical decomposition}}$

Theorem

Let $S = (X, \rightarrow_S, \leq)$ be a WSTS, then $\widehat{S} = (\widehat{X}, \rightarrow_{\widehat{S}}, \subseteq)$ such that

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Theorem

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- \widehat{S} is finitely branching,
- \widehat{S} has (strong) monotony,
- \widehat{S} is *not always* a WSTS (Jančar IPL'99).

Relating executions of S and \widehat{S}

Let $S = (X, \rightarrow_S, \leq)$ be a WSTS, then

- if $x \xrightarrow{k}_S y$,

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- if $I \xrightarrow{k}_{\widehat{S}} J$, then for every $y \in J$ there exists $x \in I$ such that $x \xrightarrow{*}_S y' \geq y$.

Relating executions of S and \widehat{S}

Let $S = (X, \rightarrow_S, \leq)$ be a WSTS with transitive monotony, then

- if $x \xrightarrow{k}_S y$, then for every ideal $I \supseteq \downarrow x$ there exists an ideal $J \supseteq \downarrow y$ such that $I \xrightarrow{k}_{\widehat{S}} J$,
- if $I \xrightarrow{k}_{\widehat{S}} J$, then for every $y \in J$ there exists $x \in I$ such that $x \xrightarrow{\geq k}_S y' \geq y$.

Relating executions of S and \widehat{S}

Let $S = (X, \rightarrow_S, \leq)$ be a WSTS with strong monotony, then

- if $x \xrightarrow{k}_S y$, then for every ideal $I \supseteq \downarrow x$ there exists an ideal $J \supseteq \downarrow y$ such that $I \xrightarrow{k}_{\widehat{S}} J$,
- if $I \xrightarrow{k}_{\widehat{S}} J$, then for every $y \in J$ there exists $x \in I$ such that $x \xrightarrow{k}_S y' \geq y$.

Theorem

Strong termination is decidable for infinitely branching WSTS with transitive monotony and such that \widehat{S} is a post-effective WSTS.

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Post-effectiveness

Possible to compute cardinality of

$$\text{Post}(\odot \circ \circ) = \circ \odot \circ, \circ \odot \odot, \circ \odot \odot \odot, \dots$$

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Proof

- Executions bounded in S iff bounded in \widehat{S} .

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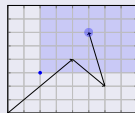
Proof

- Executions bounded in S iff bounded in \widehat{S} .
- \widehat{S} finitely branching, can decide termination in \widehat{S} by Finkel & Schnoebelen 2001.

Coverability

Input: (X, \rightarrow, \leq) a WSTS, $x_0, x \in X$.

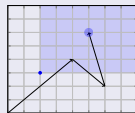
Question: $x_0 \xrightarrow{*} x' \geq x$?



Coverability

Input: (X, \rightarrow, \leq) a WSTS, $x_0, x \in X$.

Question: $x_0 \in \uparrow \text{Pre}^*(\uparrow x)$?



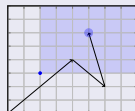
Backward method (Abdulla, Cerans, Jonsson & Tsay IC'00)

Compute $\uparrow \text{Pre}^*(\uparrow x)$ iteratively assuming $\uparrow \text{Pre}(U)$ computable.

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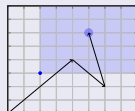
Question: $x \in \downarrow \text{Post}^*(x_0)$?



Coverability

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Question: $x \in \downarrow \text{Post}^*(x_0)$?



Forward method

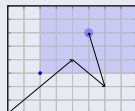
Coverability:

- Enumerate executions $\downarrow x_0 \xrightarrow{*} \widehat{S} \ I$,
- Accept if $x \in I$.

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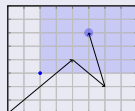
Non coverability:

- Enumerate $D \subseteq X$ downward closed, $x_0 \in D$ and $\downarrow \text{Post}_{\mathcal{S}}(D) \subseteq D$
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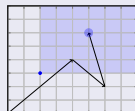
Non coverability:

- Enumerate $D = I_1 \cup \dots \cup I_k$
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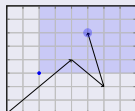
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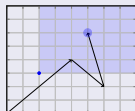
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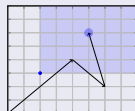
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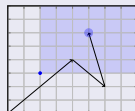
Non coverability:

- Enumerate $D \subseteq X$ downward closed, $\exists j$ s.t. $\downarrow x_0 \subseteq I_j$
- Reject if $x \notin D$.

Coverability

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Forward method

Coverability:

- Enumerate executions $\downarrow x_0 \xrightarrow{*}_S I$,
- Accept if $x \in I$.

Non coverability:

- Enumerate $D \subseteq X$ downward closed, $x_0 \in D$ and $\downarrow \text{Post}_S(D) \subseteq D$
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Further results for infinitely branching WSTS

- Boundedness is decidable for post-effective WSTS with strict monotony,

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- Strong maintainability is decidable for WSTS with strong monotony and such that \hat{S} is a post-effective WSTS.

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- \exists general class of infinitely branching WSTS with a Karp-Miller procedure?

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- Toward the algorithmics of complete WSTS.
- What else can we do with the WSTS completion?

Thank you!