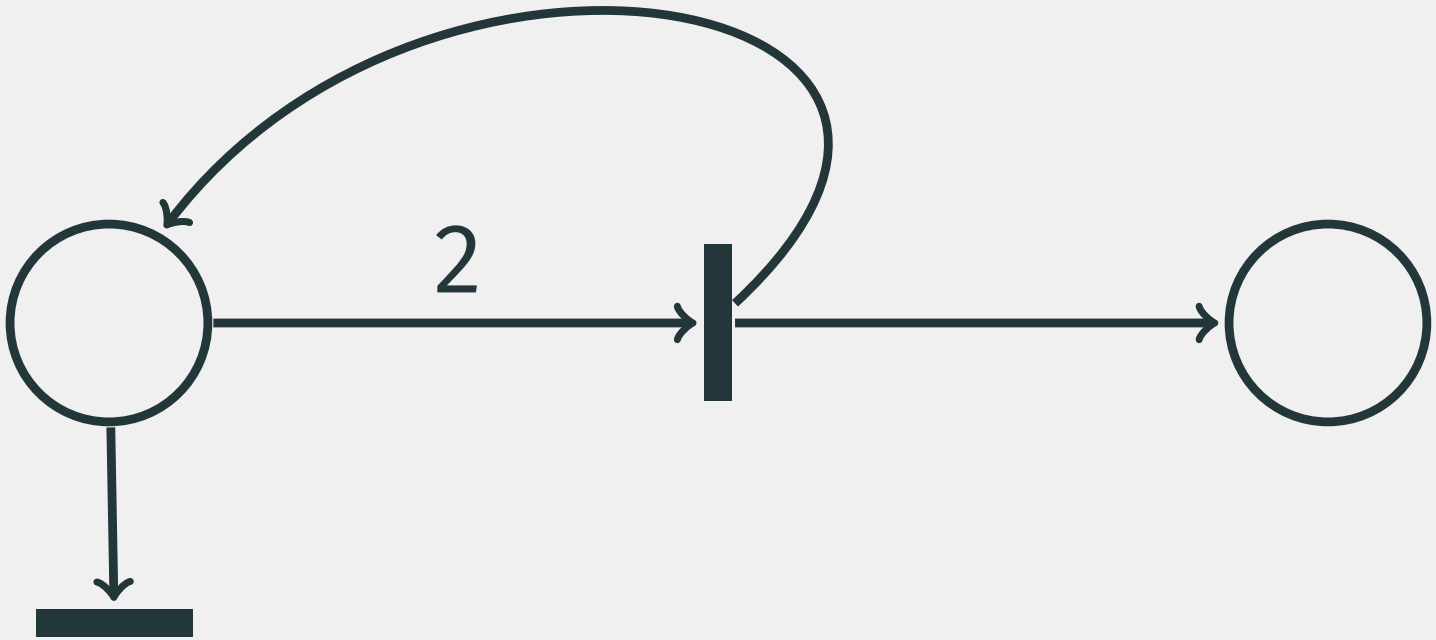
Approaching the Coverability Problem Continuously

Michael Blondin, Alain Finkel, Christoph Haase, Serge Haddad

(Discrete) Petri nets



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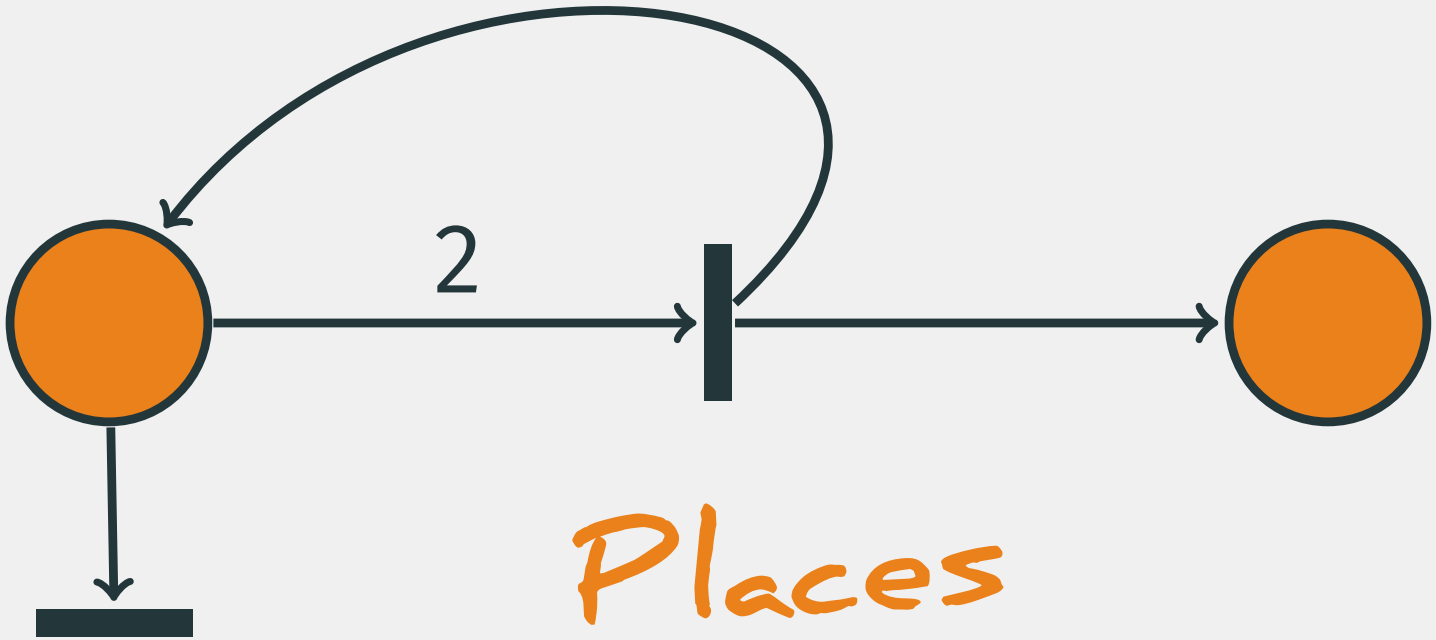

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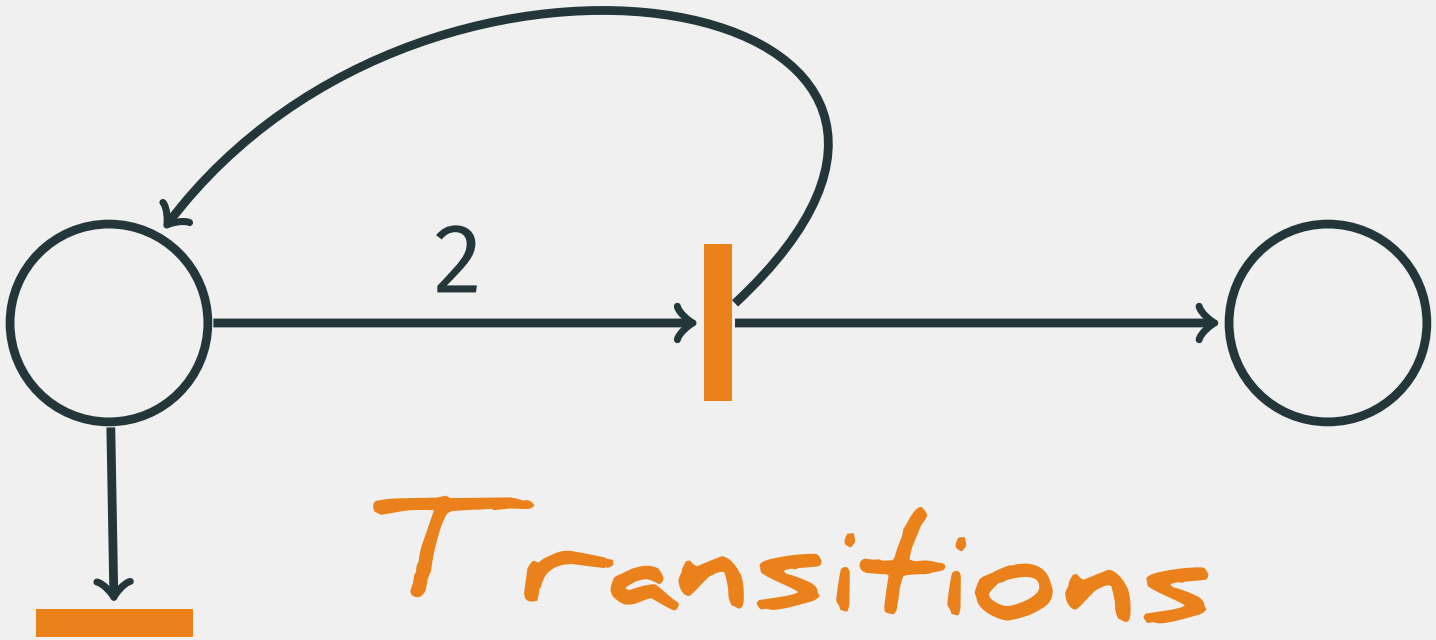

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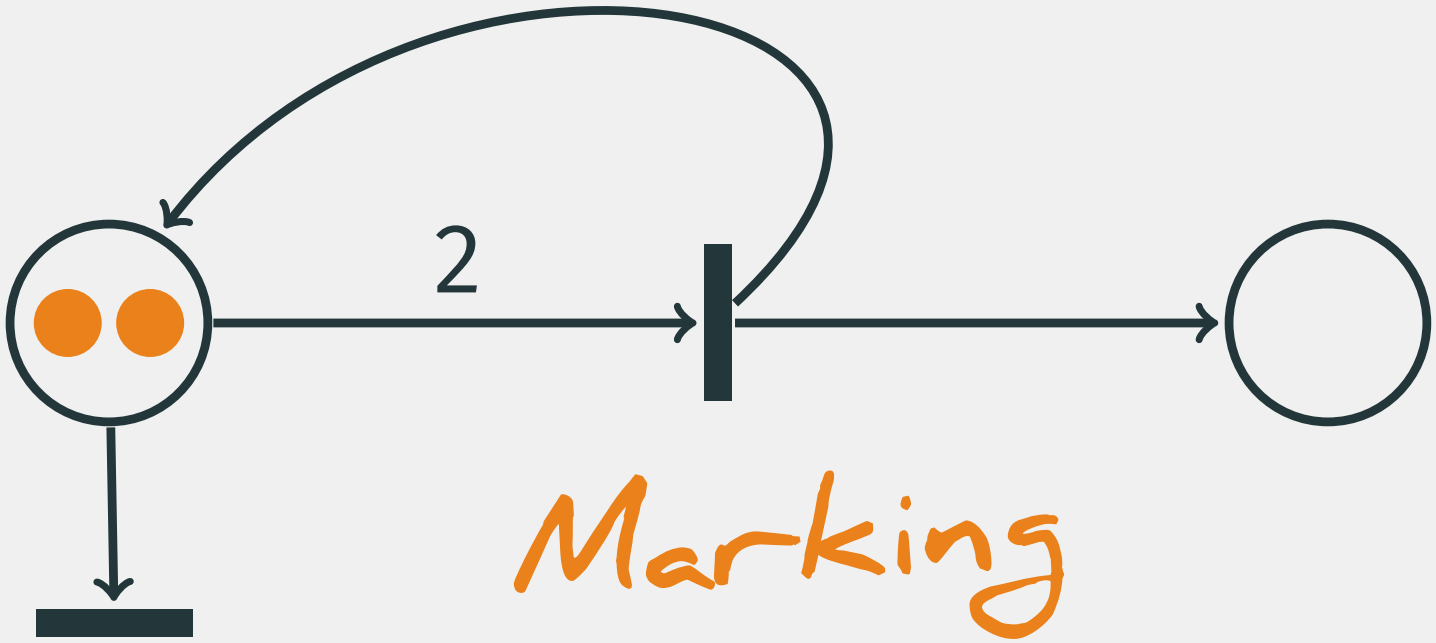

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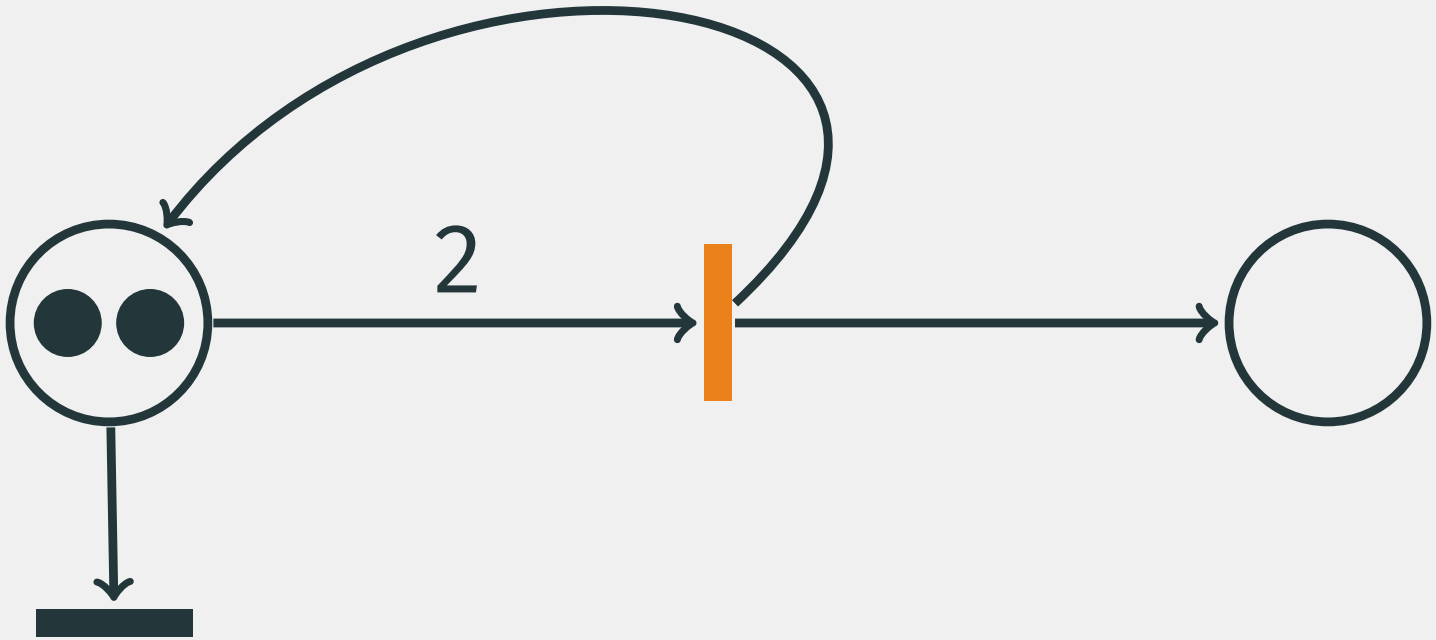

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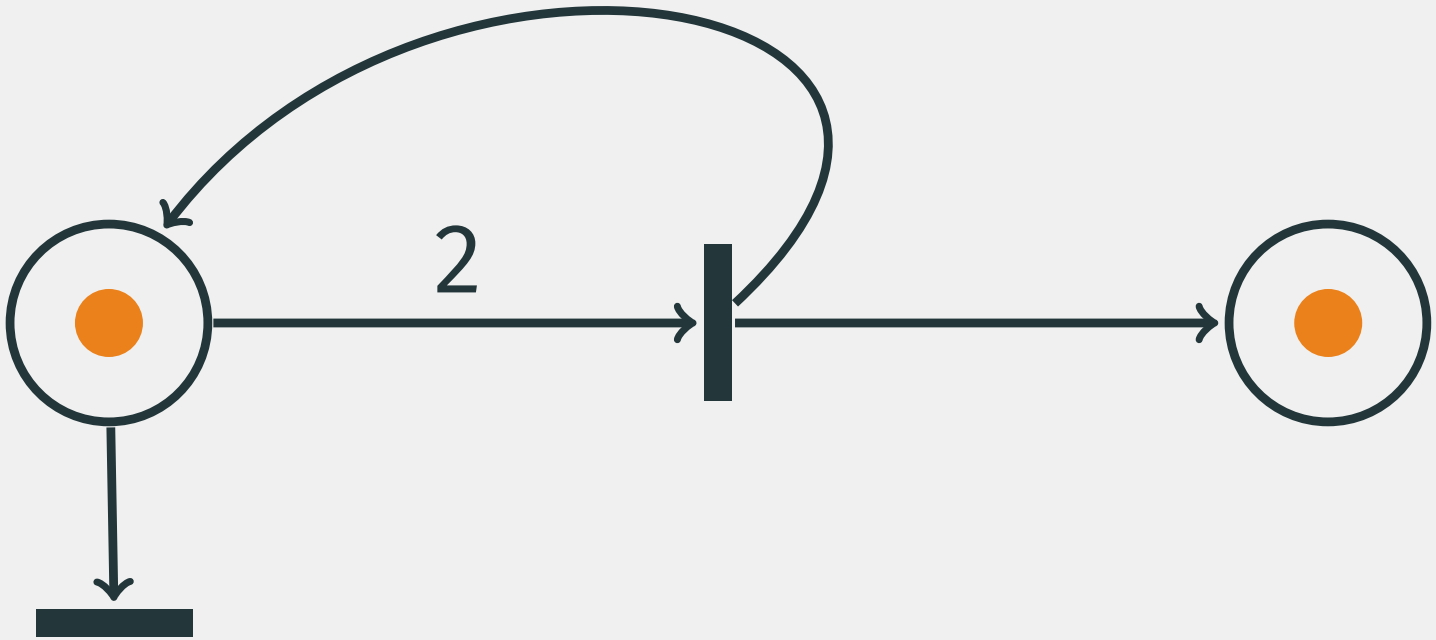

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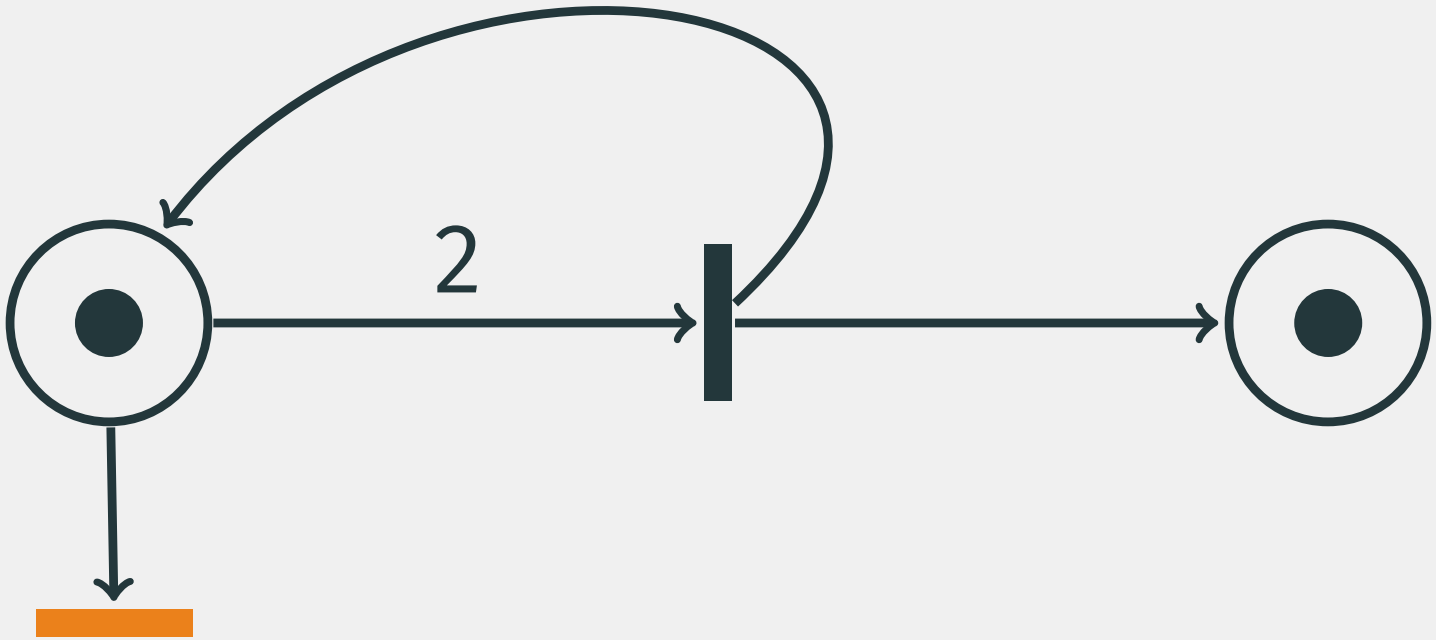

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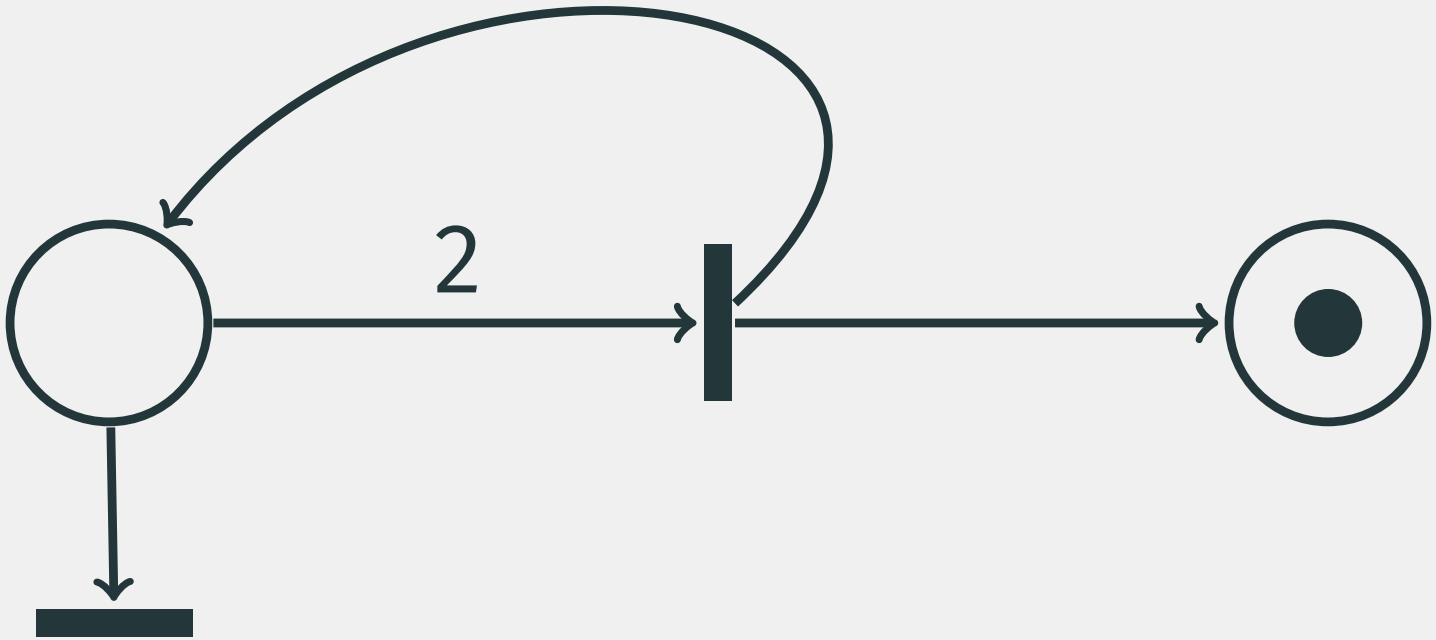

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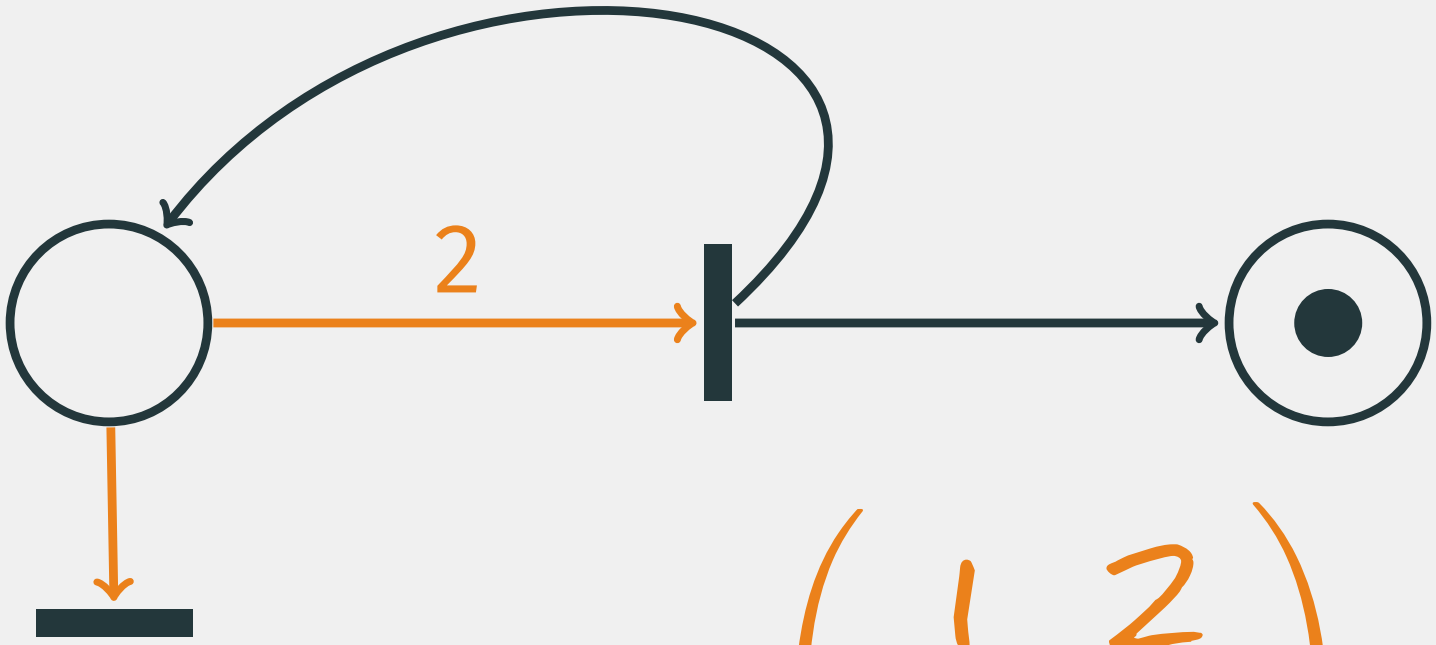

$$Pre = \begin{pmatrix} 1 & 2 \\ 0 & 0 \end{pmatrix}$$

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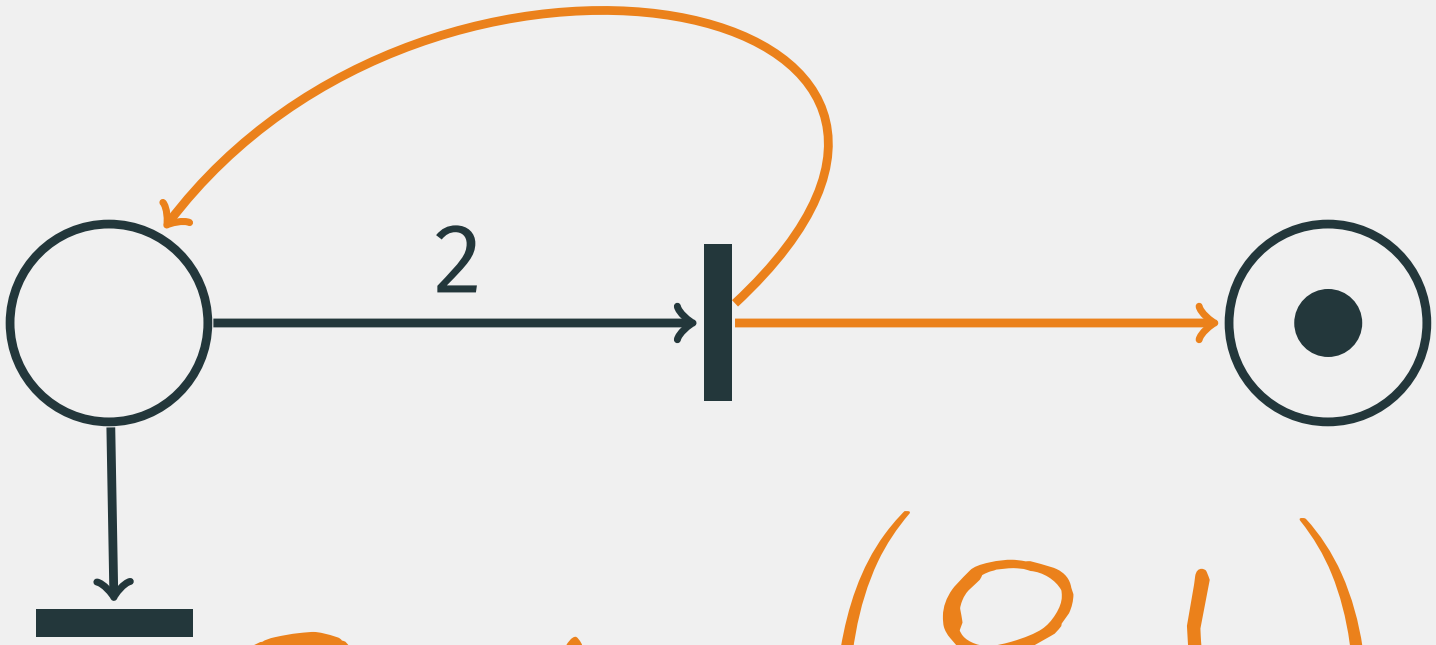

$$\mathrm{Post} = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}$$

Verifying safety with Petri nets

Lamport mutual exclusion "1-bit algorithm"

Verifying safety with Petri nets

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Verifying safety with Petri nets

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while True:
  x = True
  while y: pass
  # critical section
  x = False
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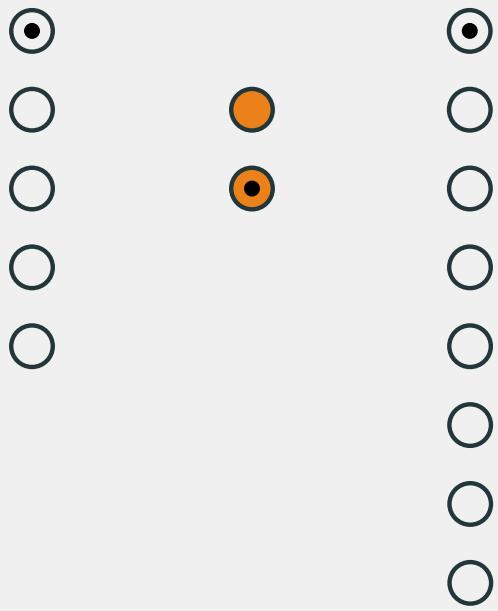

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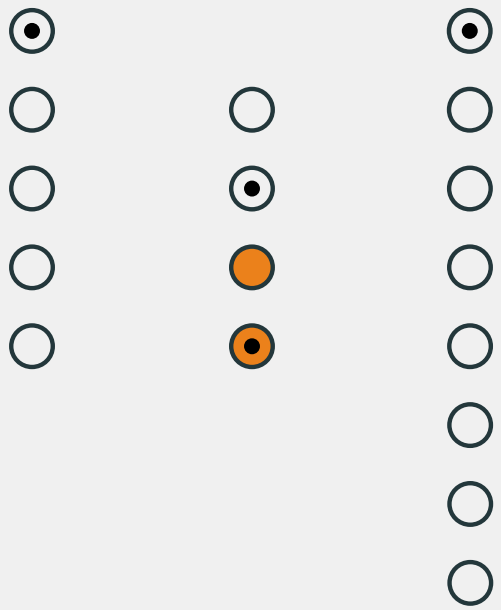

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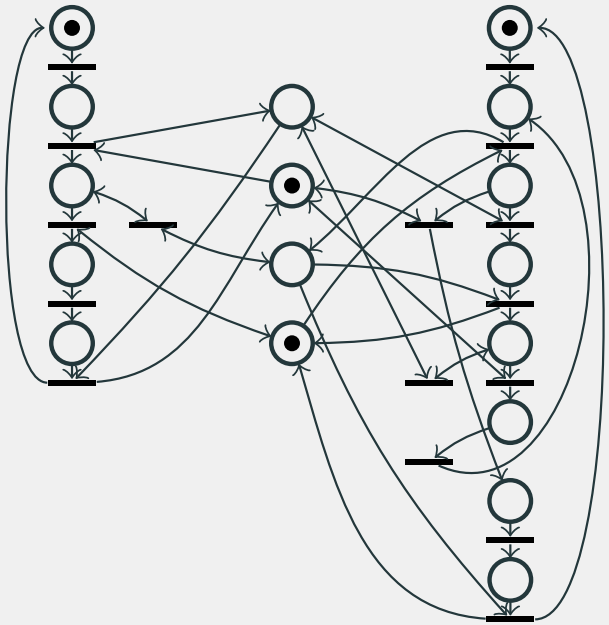

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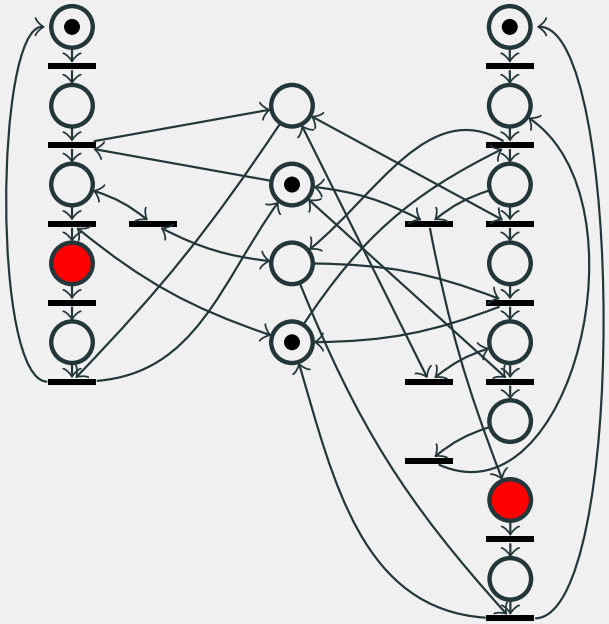

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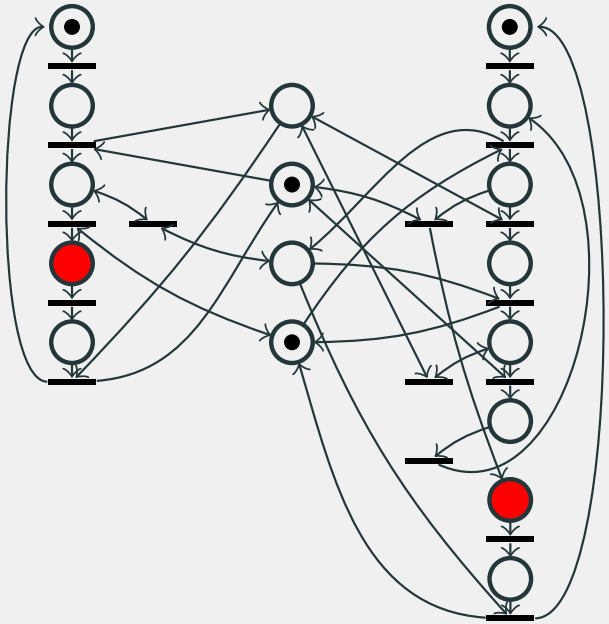
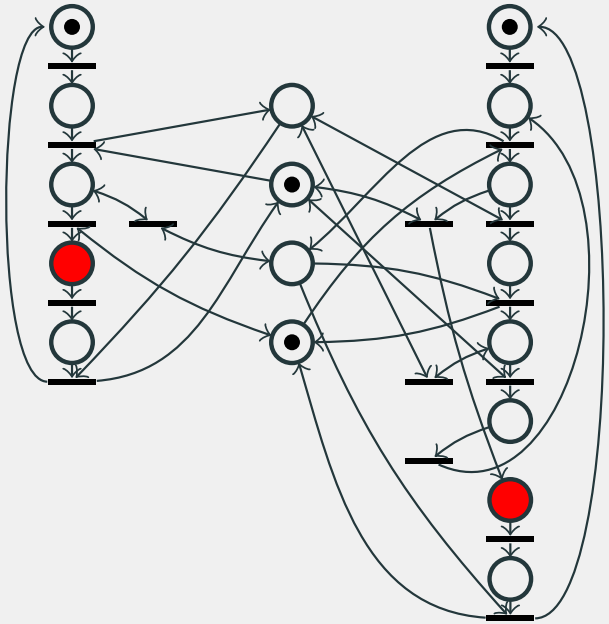


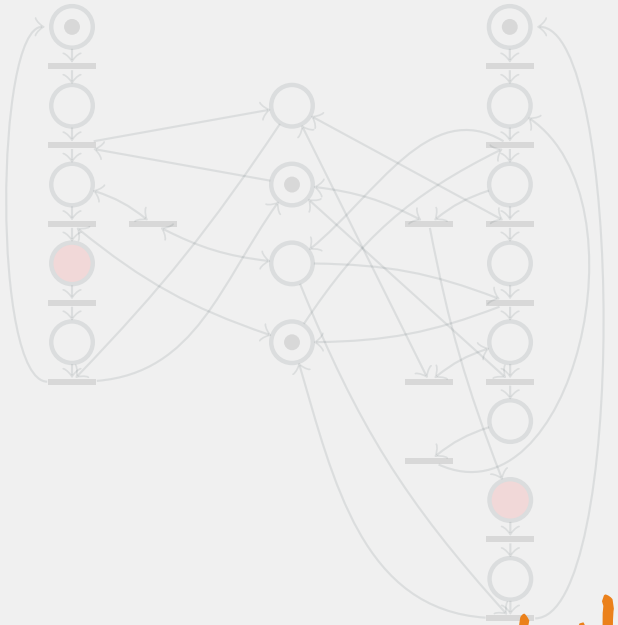
Processes at both critical sections $\iff$ each  ≥ 1

Verifying safety with Petri nets



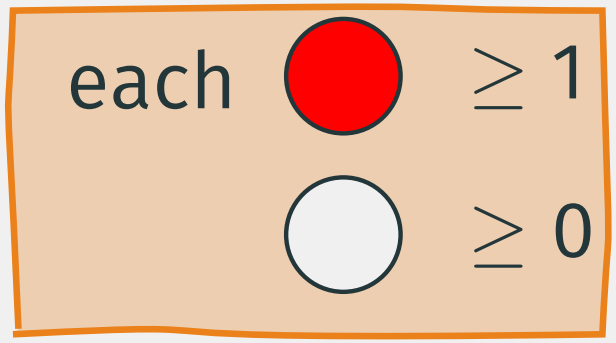
Processes at both critical sections $\iff$ each  ≥ 1,  ≥ 0

Verifying safety with Petri nets



Coverability problem

Processes at both critical sections $\Longleftrightarrow$



Coverability problem

Problem

Input:	Petri net $\mathcal{N}$, initial marking $\boldsymbol{m}_0$, target marking $\boldsymbol{m}$
Question:	Is some $\boldsymbol{m}' \geq \boldsymbol{m}$ reachable from $\boldsymbol{m}_0$ in $\mathcal{N}$?

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How to solve it?

- Forward: build reachability tree from initial marking
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- EXPSPACE-complete

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How to solve it? **Karp & Miller '69**

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How to solve it? **Arnold & Latteux '78, Abdulla *et al.* '96**

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How to solve it? Lipton '76, Rackoff '78

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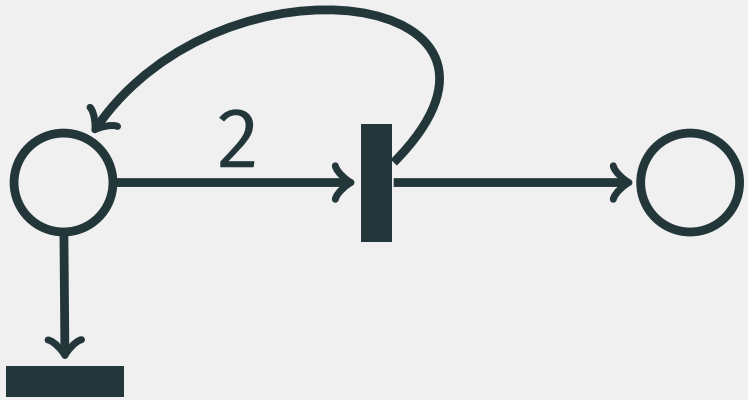
Backward algorithm



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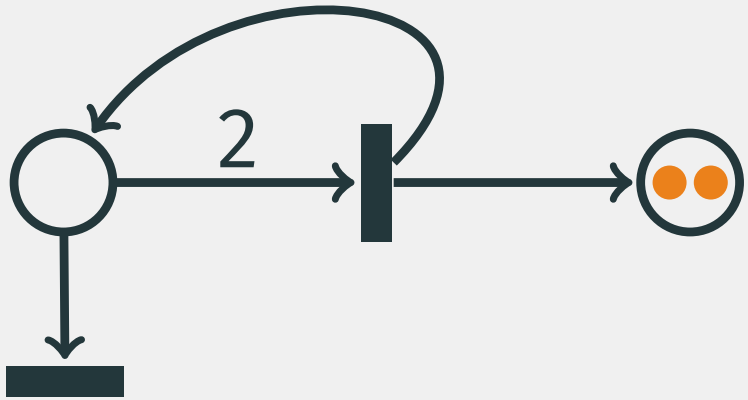
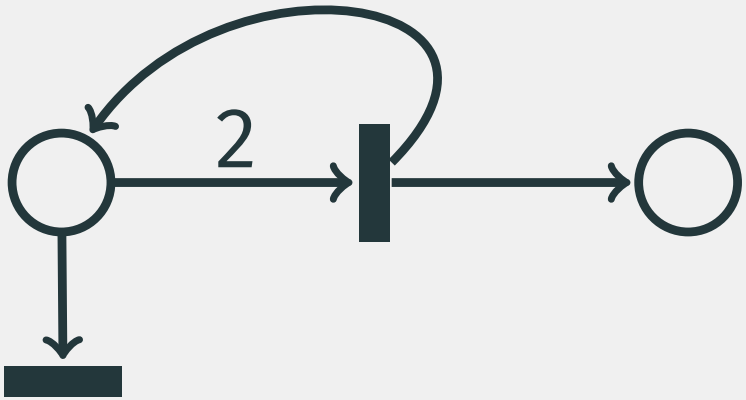
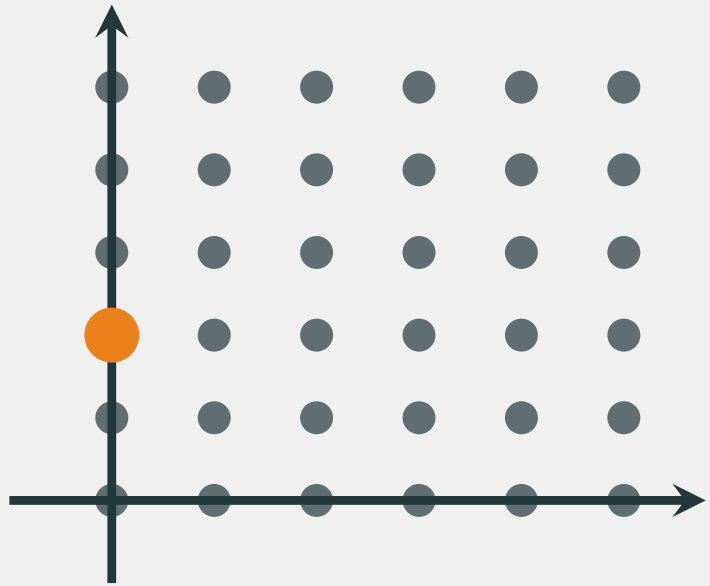


What initial markings may cover $(0, 2)$?

Backward algorithm





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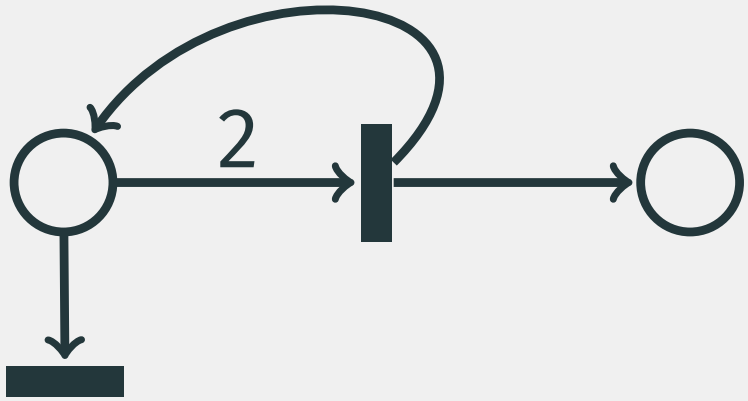

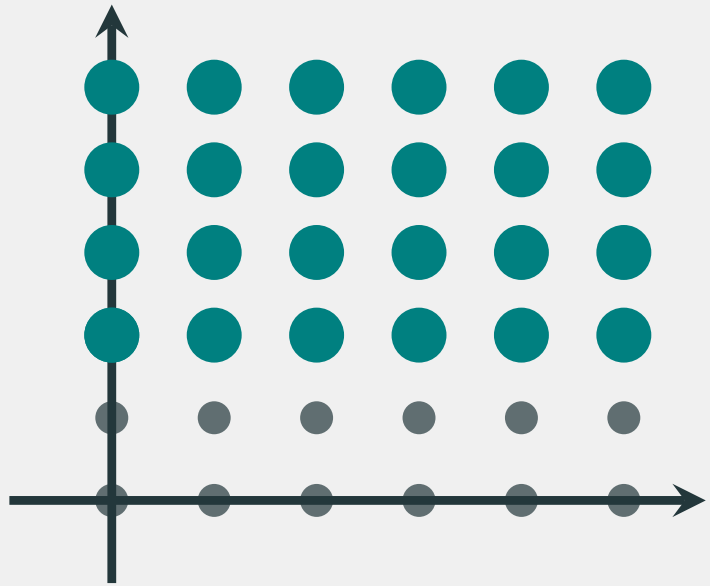

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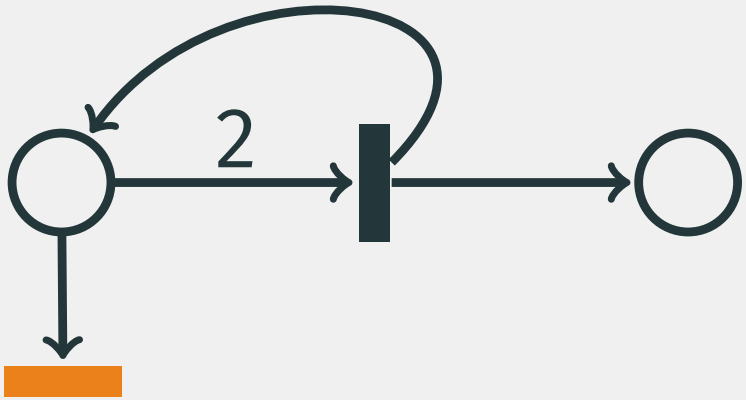

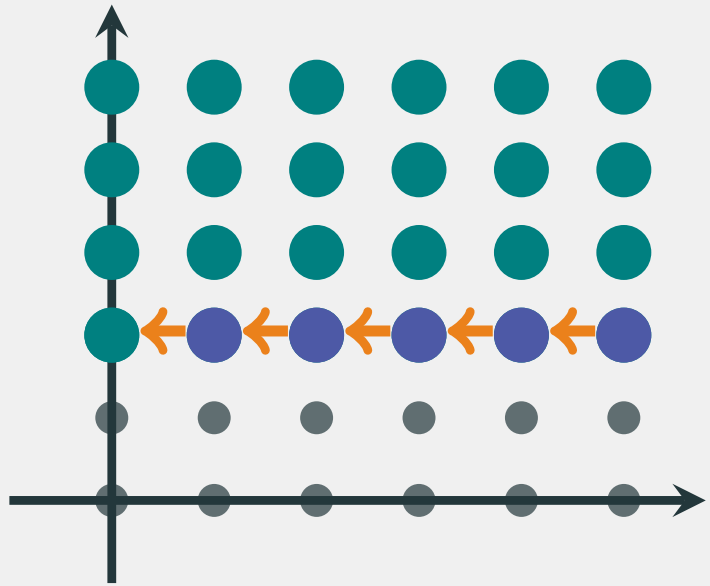

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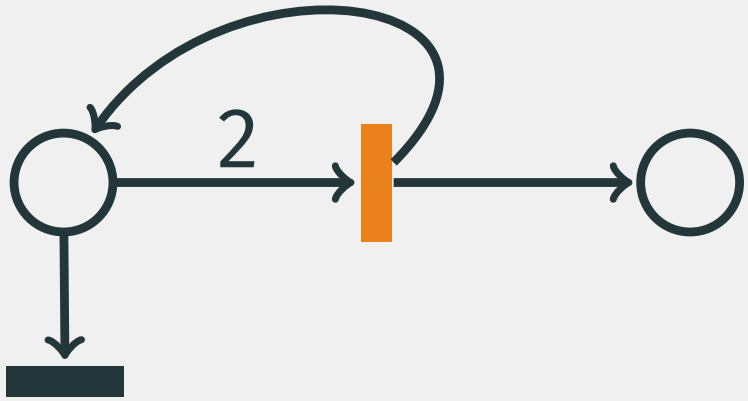

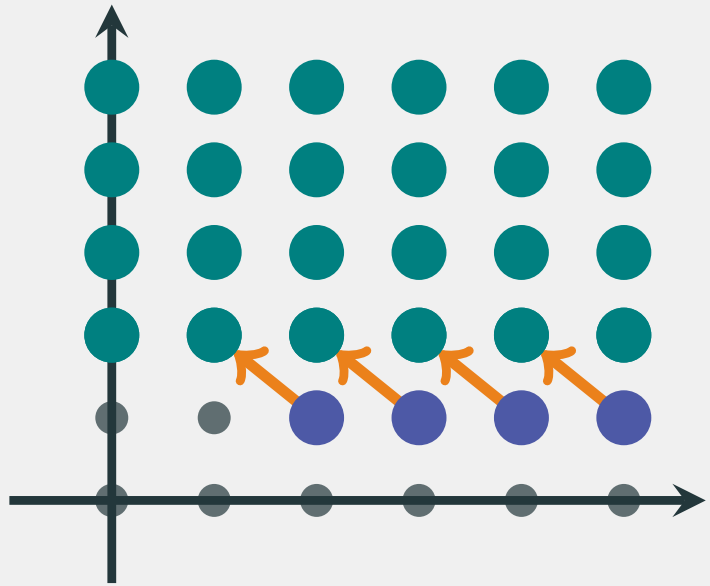

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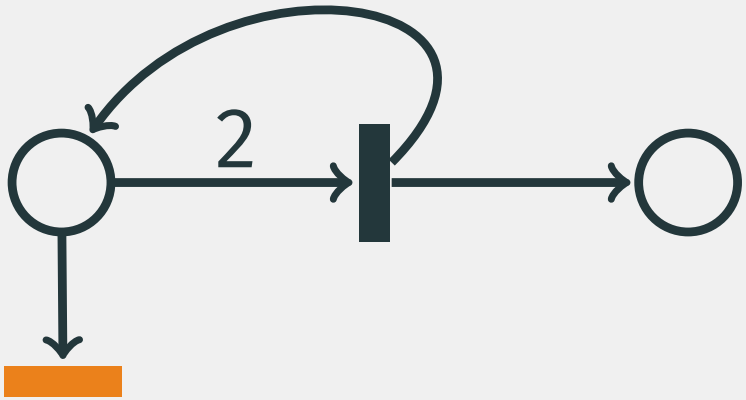

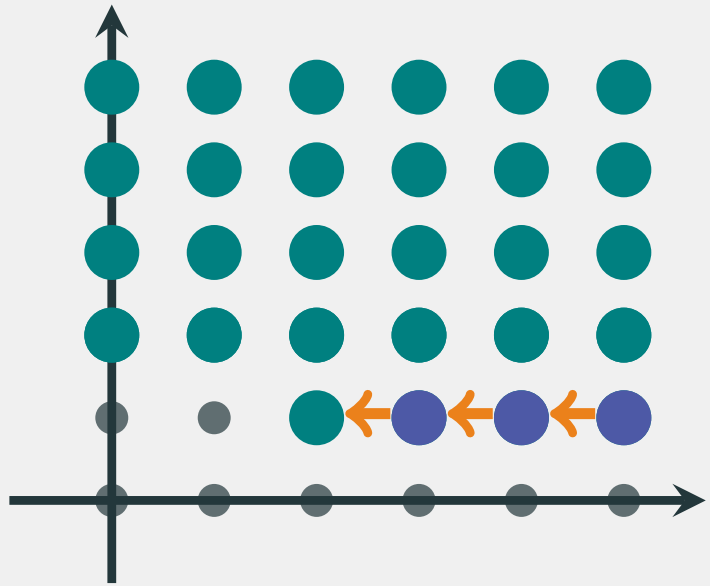

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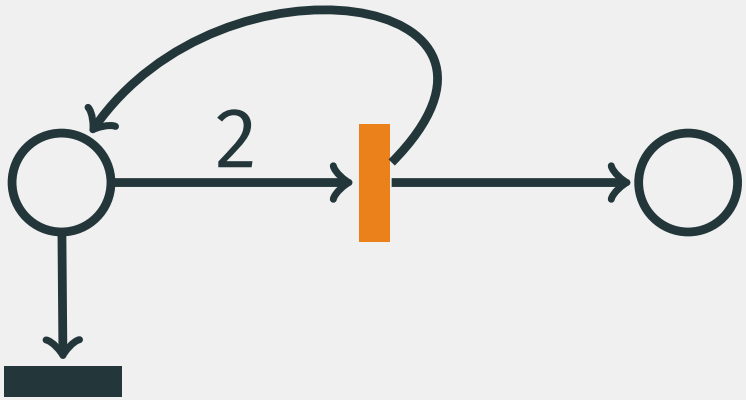

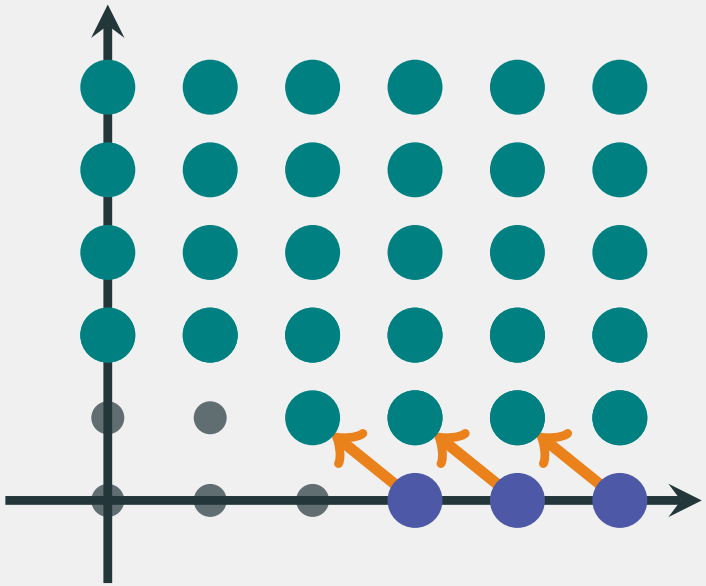

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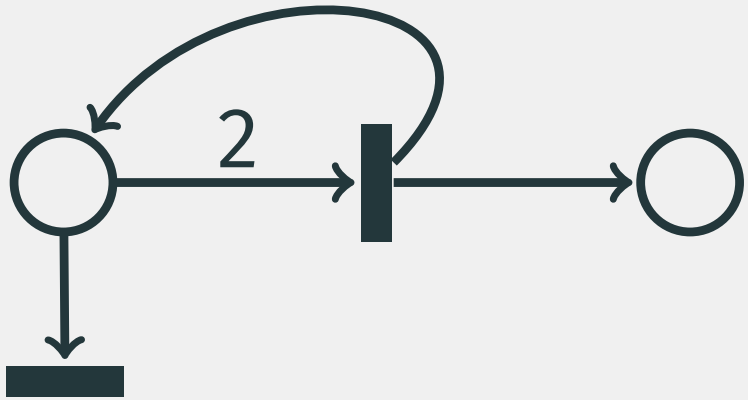
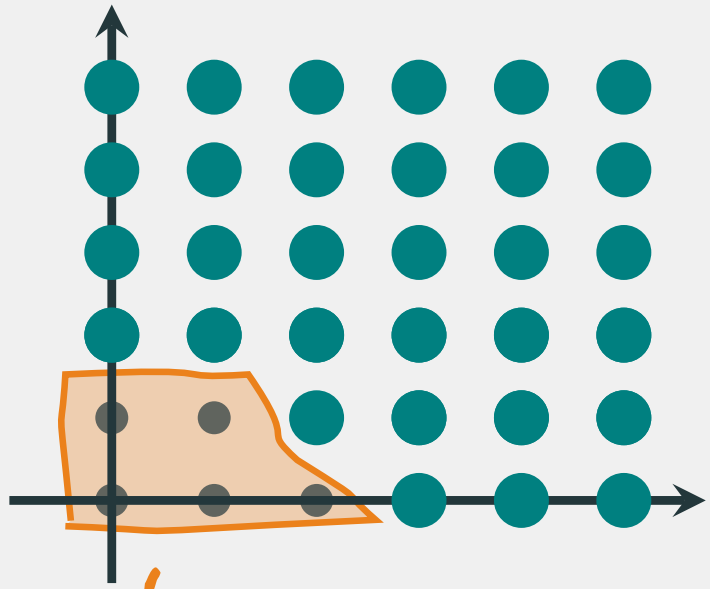
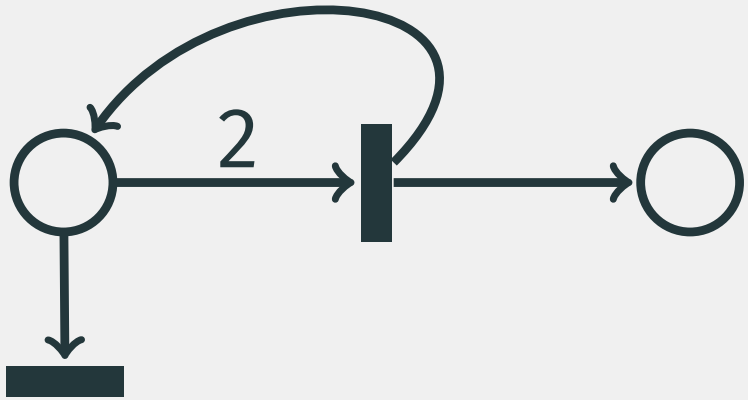
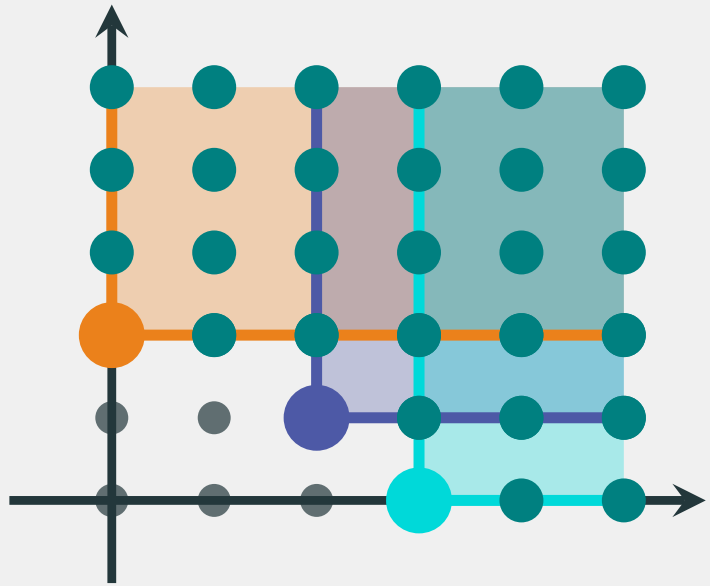




Cannot cover target marking

Backward algorithm





Basis size may become doubly exponential

(Bozzelli & Ganty '11)

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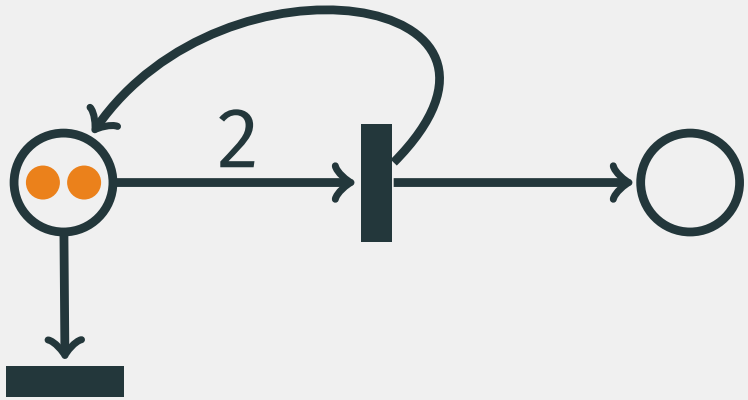

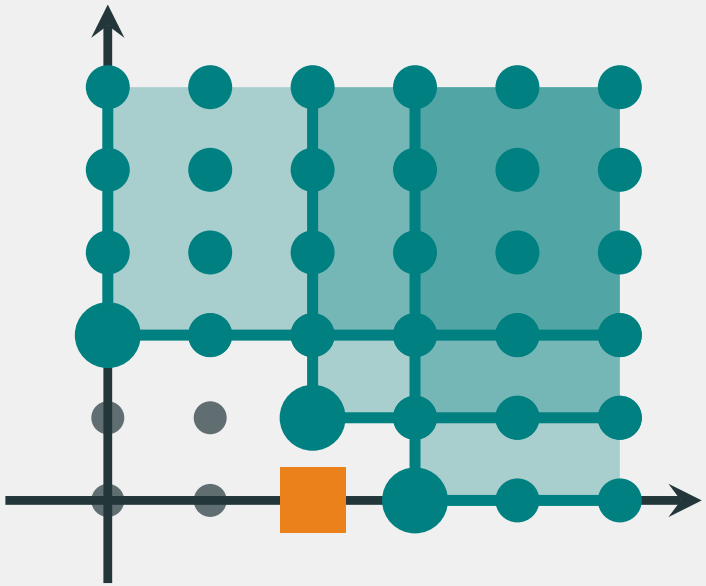

We only care about some initial marking...

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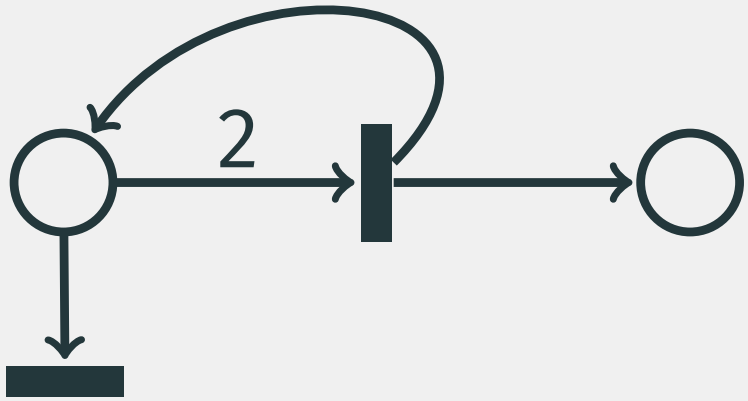

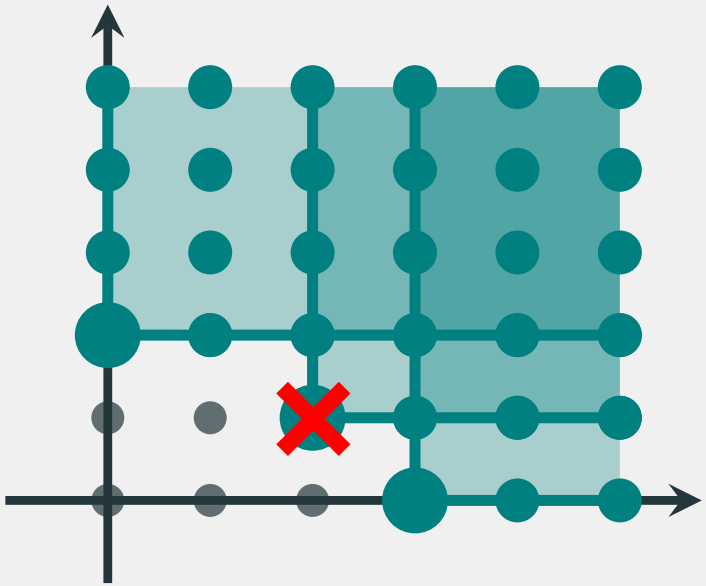

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Speedup by pruning basis!

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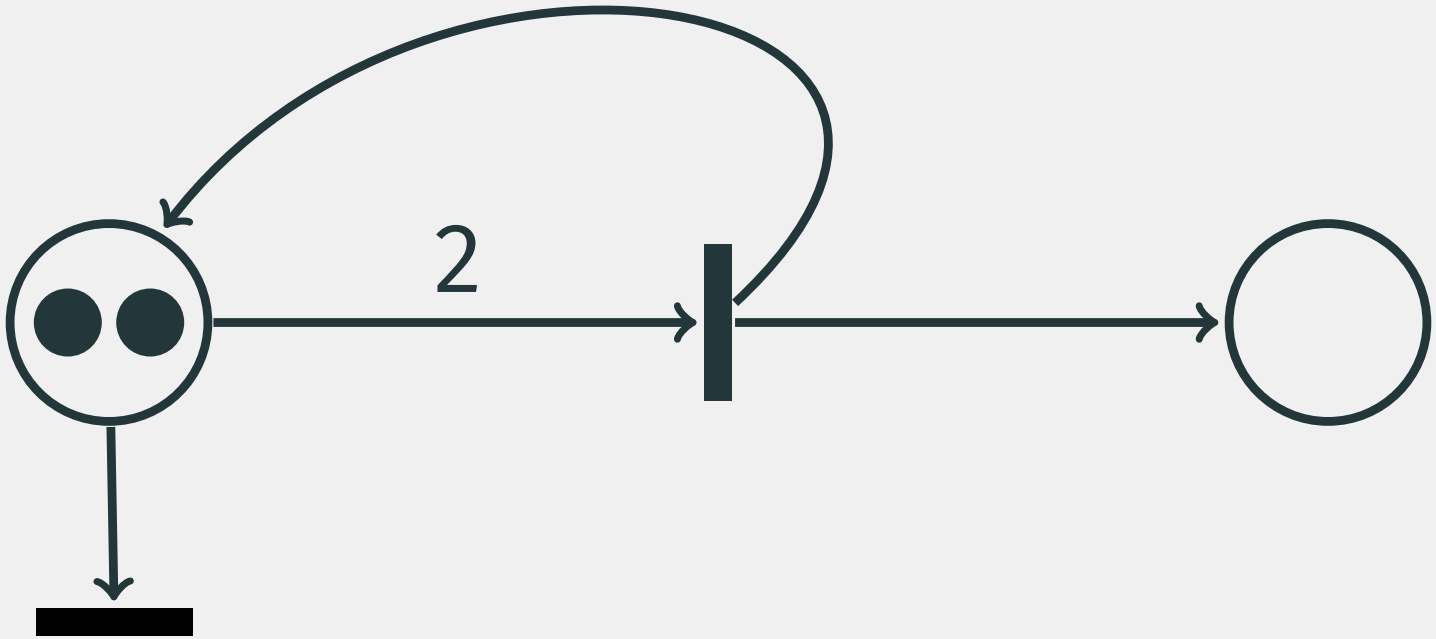

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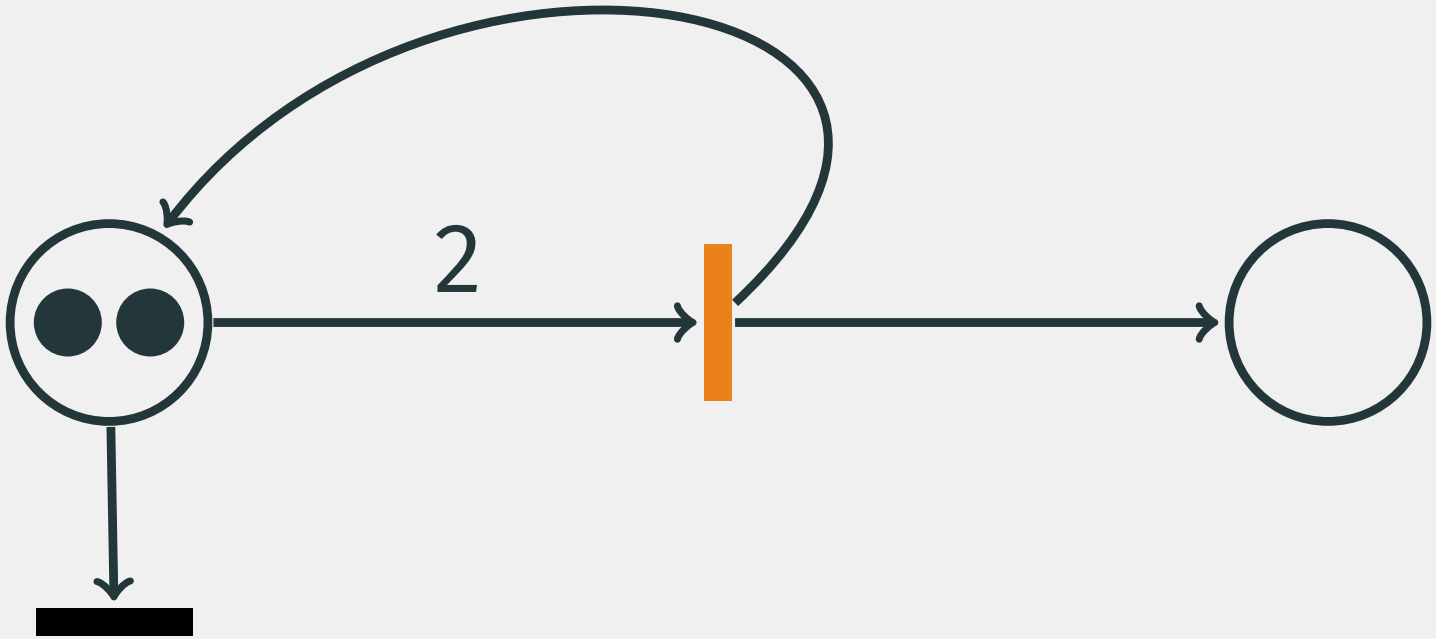

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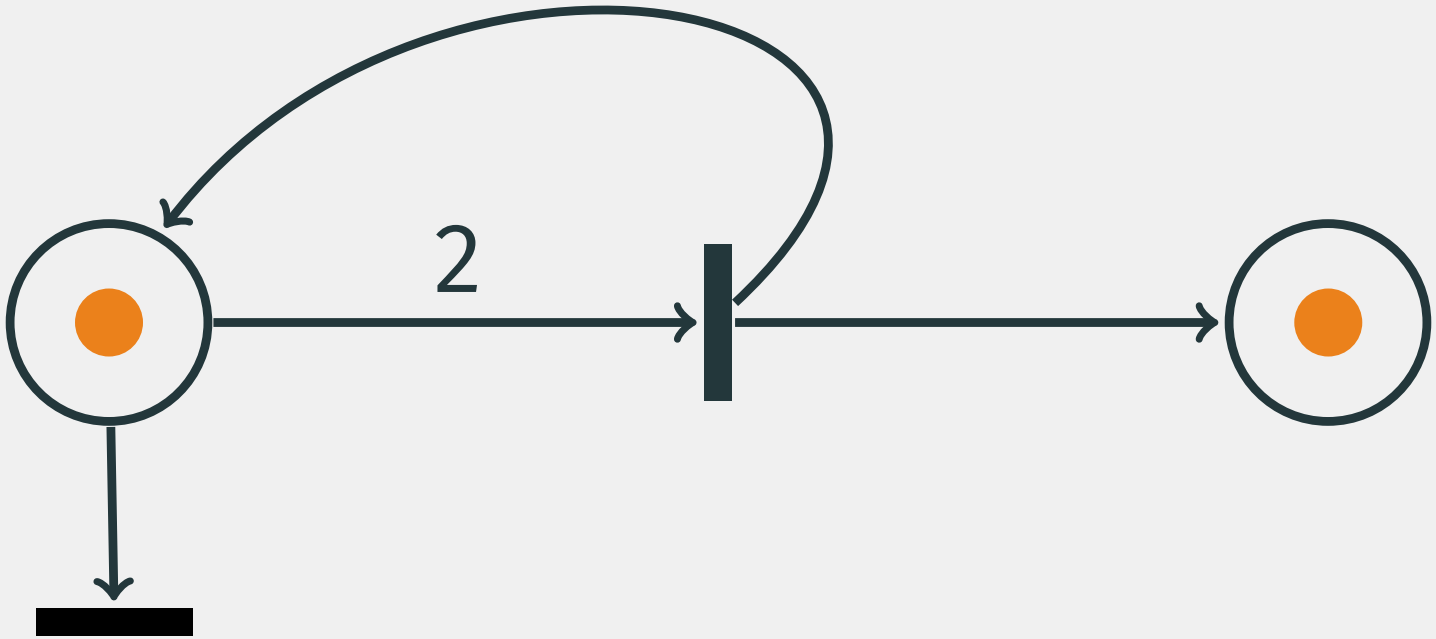

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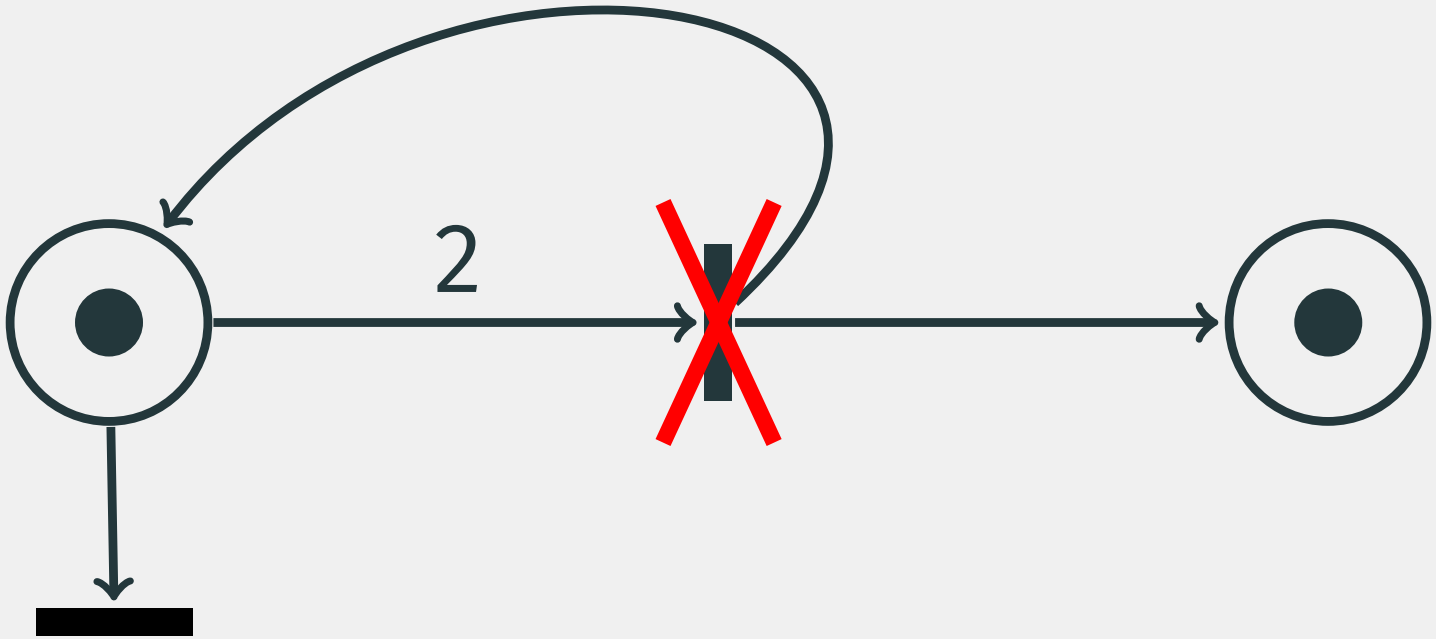

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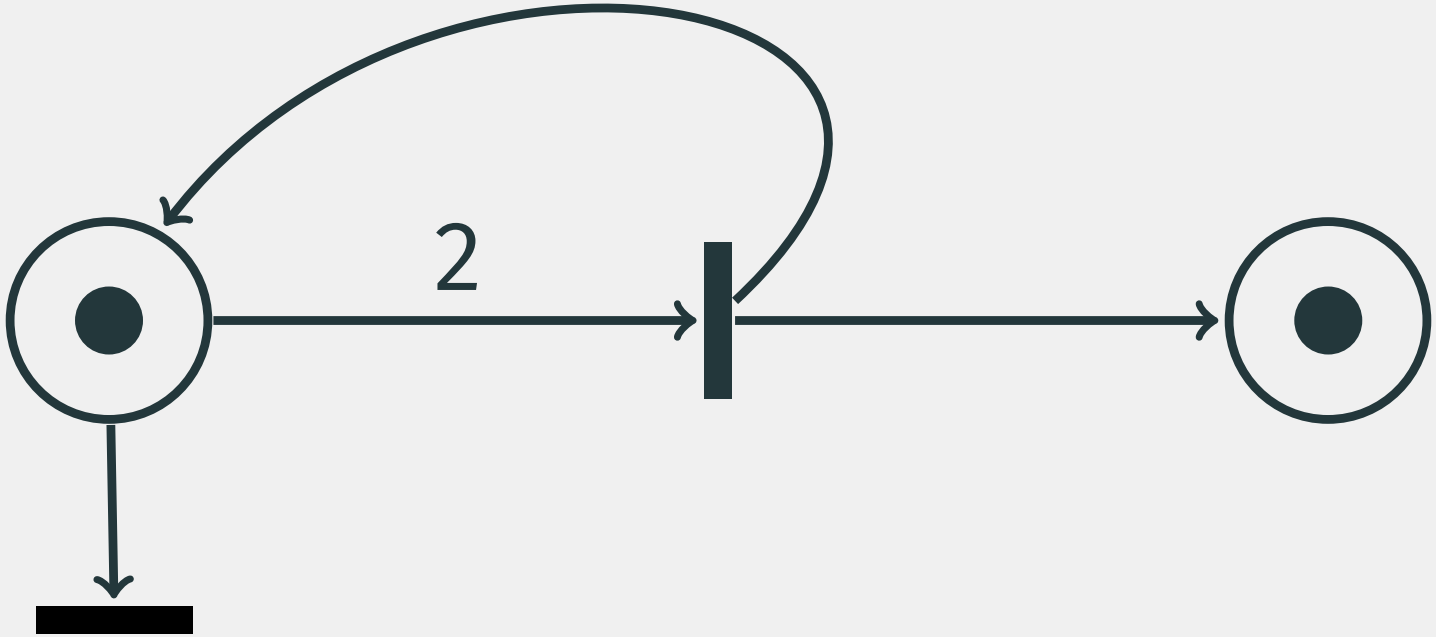

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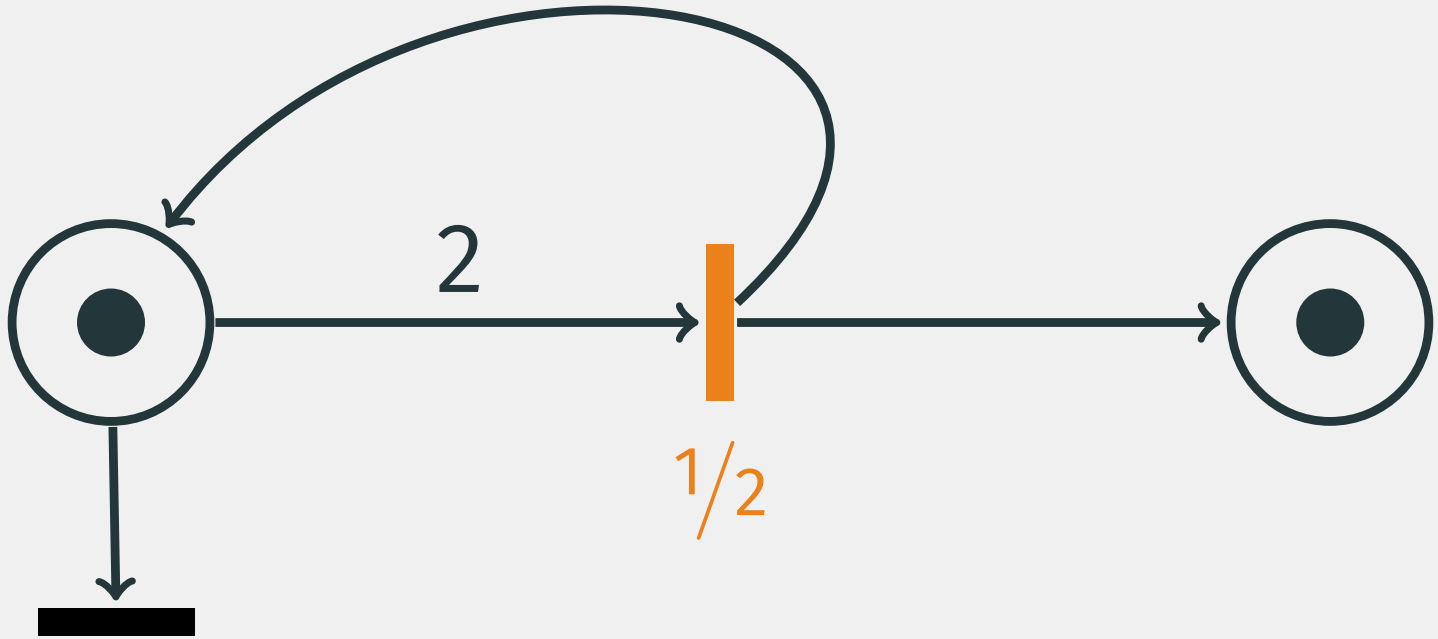

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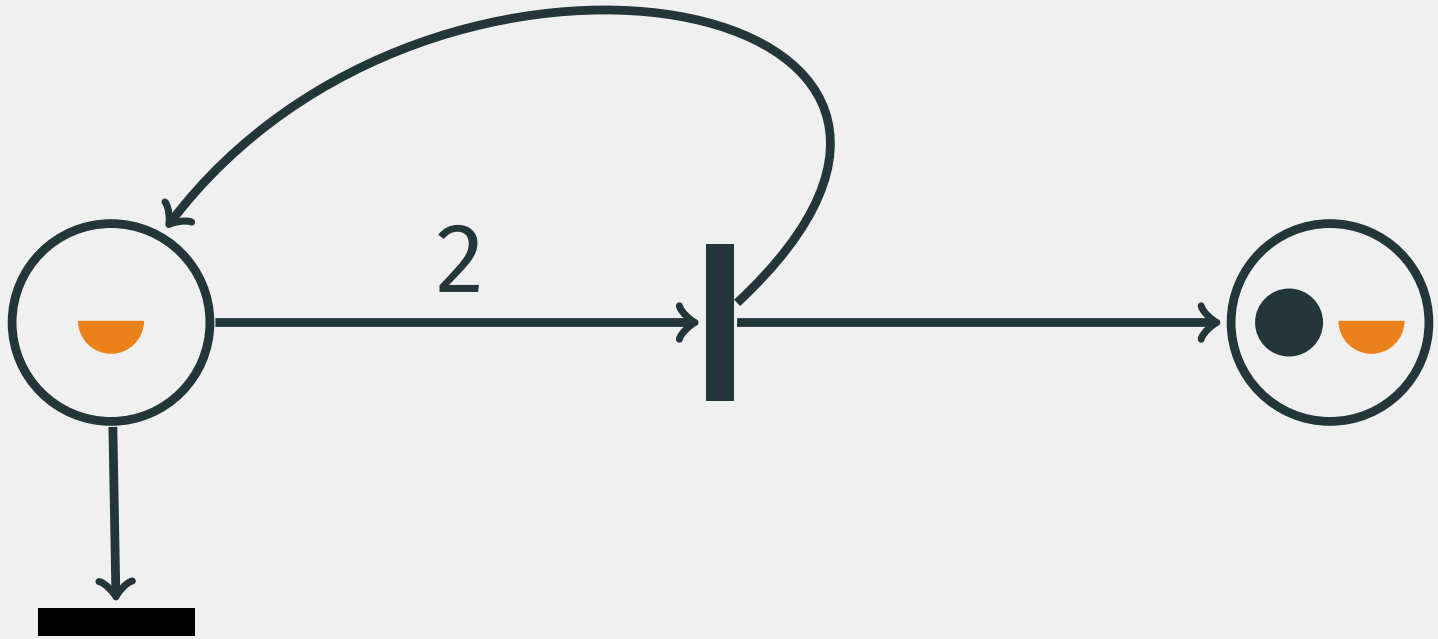

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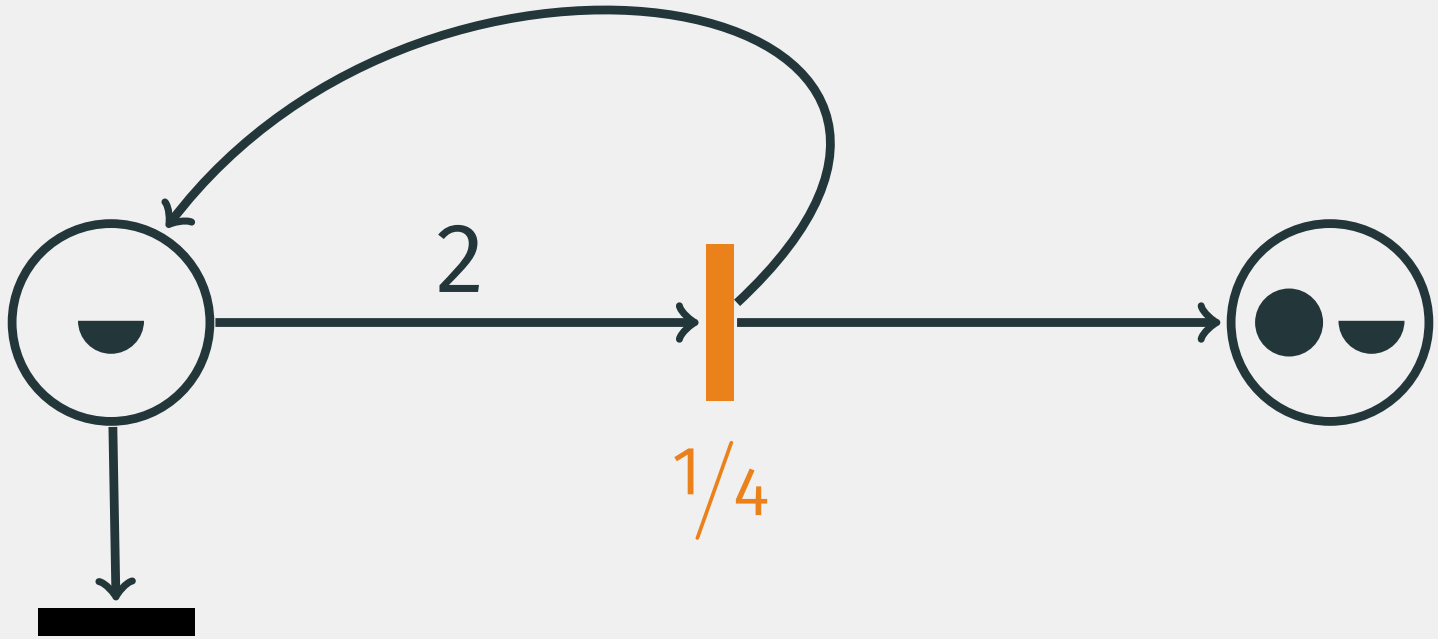

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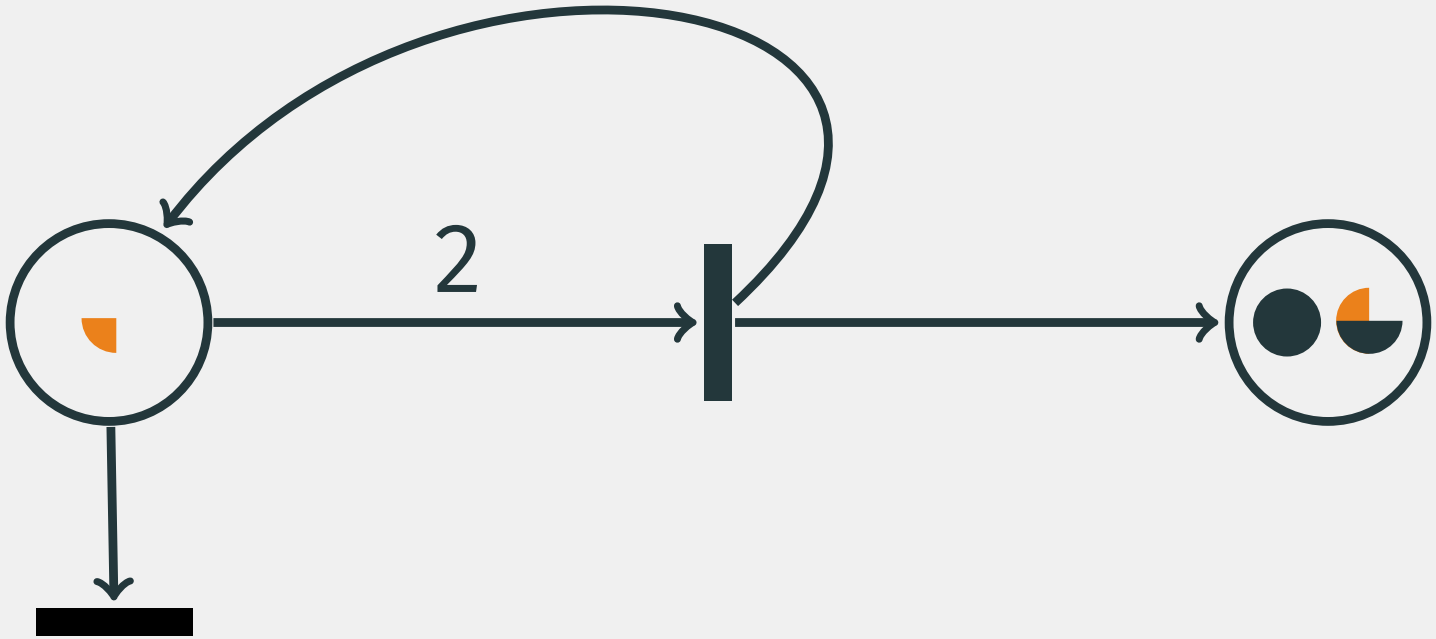

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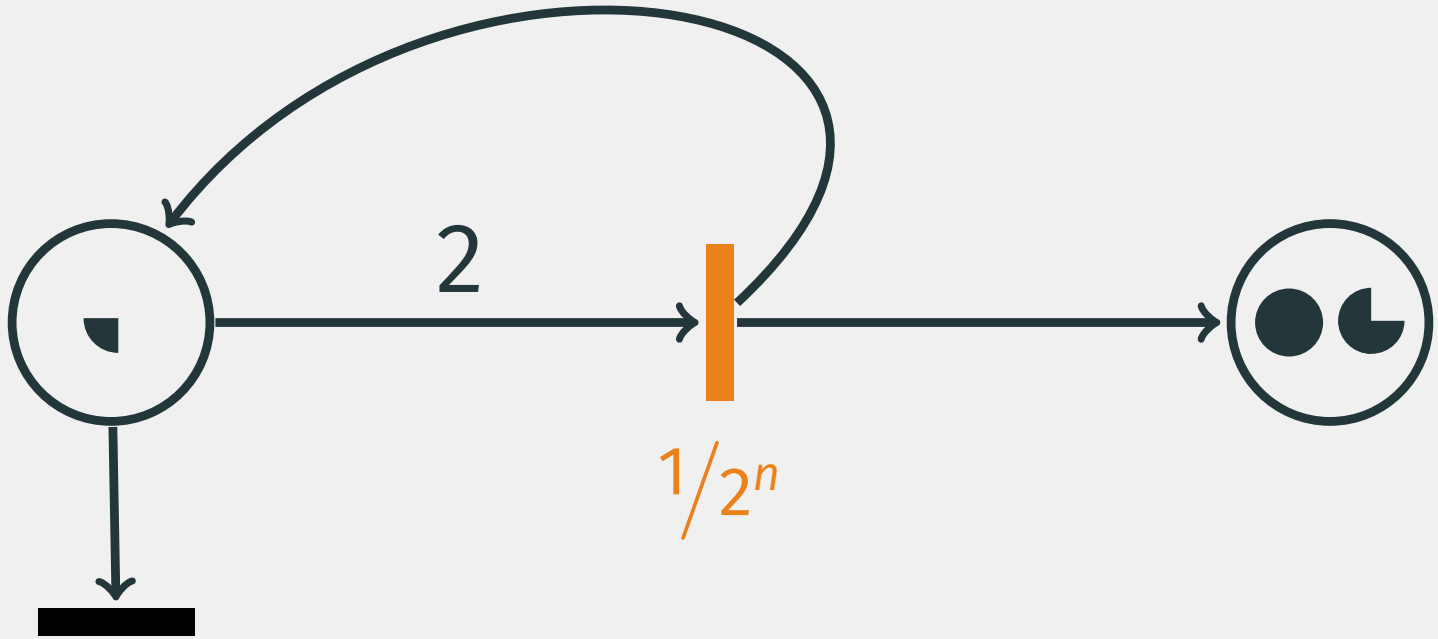

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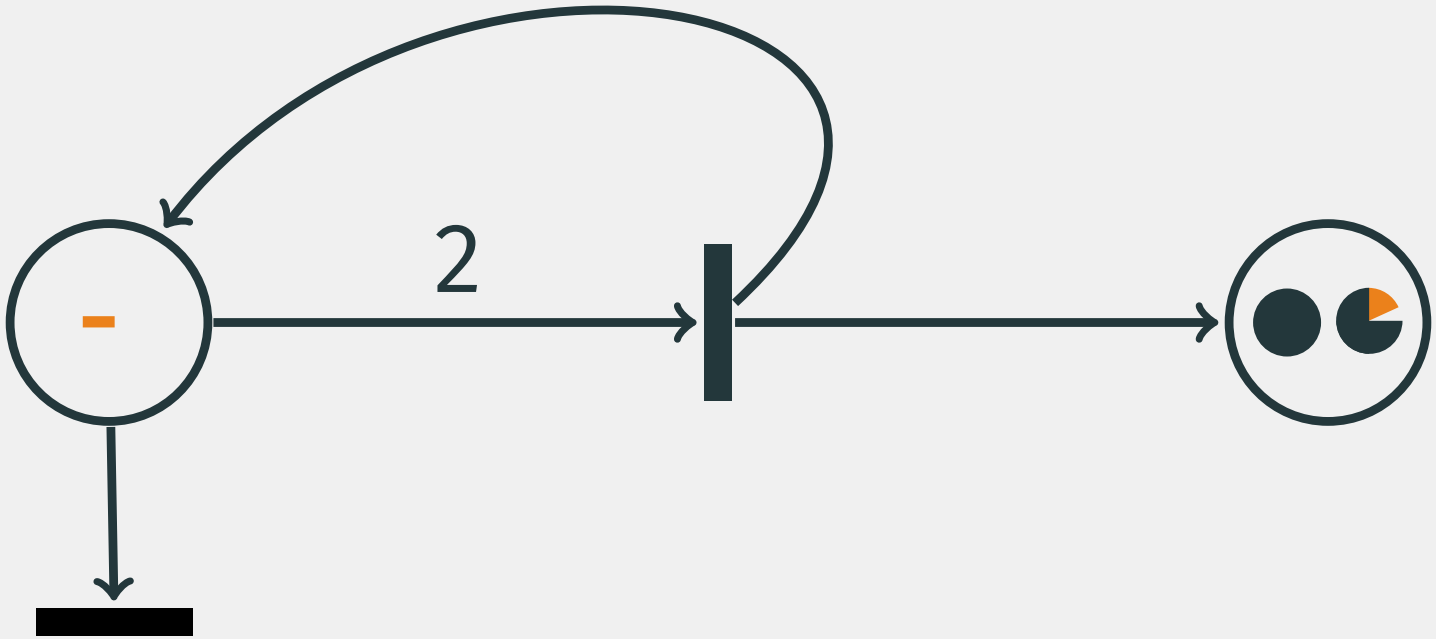

Continuity to over-approximate coverability

$\boldsymbol{m}$ is coverable from $\boldsymbol{m}_0$

$$\Downarrow$$

$\boldsymbol{m}$ is $\mathbb{Q}$-coverable from $\boldsymbol{m}_0$

Continuity to over-approximate coverability

$\boldsymbol{m}$ is coverable from $\boldsymbol{m}_0$

EXPSPACE

$\boldsymbol{m}$ is $\mathbb{Q}$-coverable from $\boldsymbol{m}_0$

PTIME

$\boldsymbol{m}_0$ and $\boldsymbol{m}$ satisfy conditions of
Esparza, Ledesma-Garza, Majumdar, Meyer & Niksic '14

PTIME / NP / coNP

Continuity to over-approximate coverability

$\boldsymbol{m}$ is not coverable from $\boldsymbol{m}_0$

Safety

$\boldsymbol{m}$ is not $\mathbb{Q}$-coverable from $\boldsymbol{m}_0$

Coverability in continuous Petri nets

Fix some continuous Petri net $(P, T, \mathbf{Pre}, \mathbf{Post})$

m is $\mathbb{Q}$-coverable from m_0 iff... **Fraca & Haddad '13**

Coverability in continuous Petri nets

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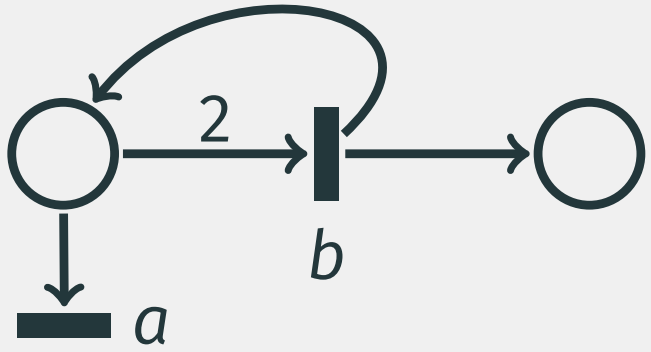
Coverability in continuous Petri nets



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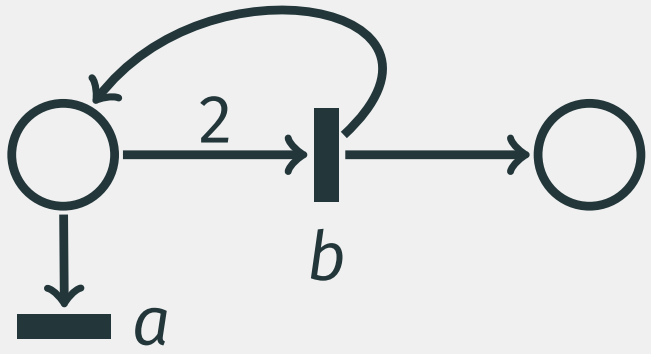

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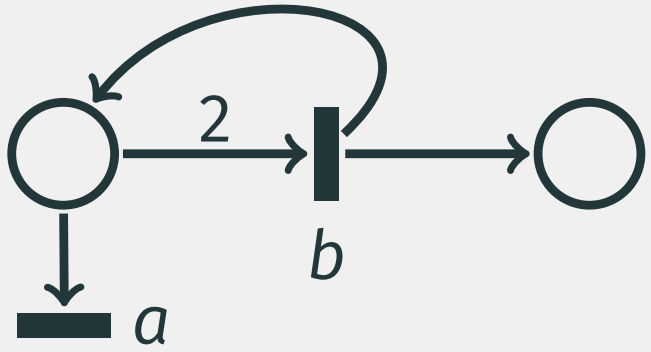

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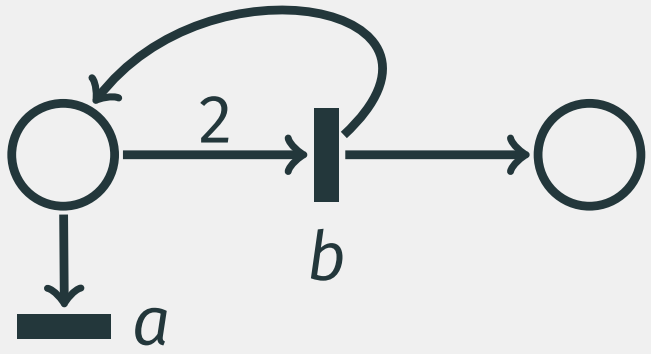
Coverability in continuous Petri nets



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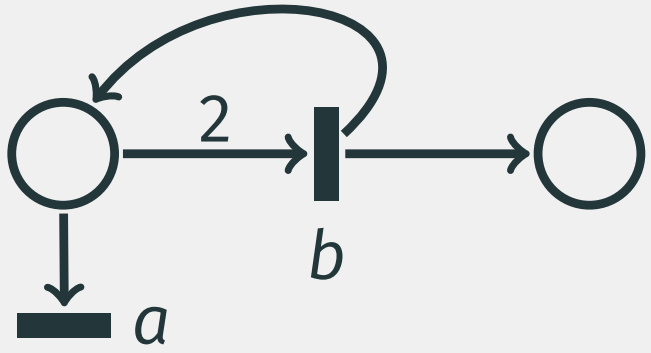

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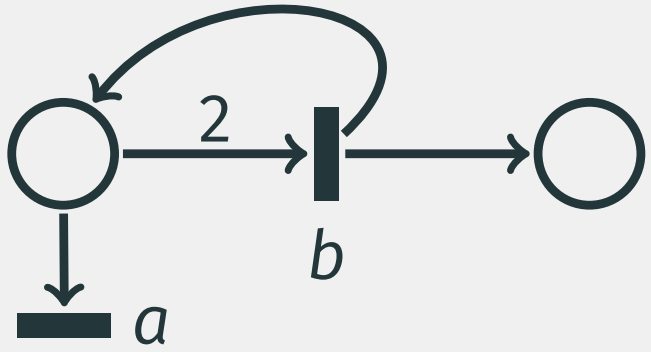

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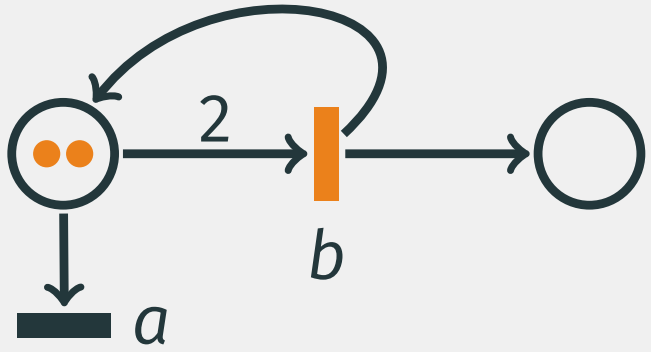
Coverability in continuous Petri nets



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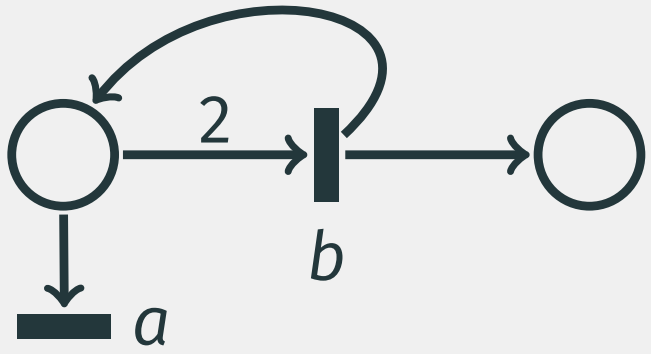

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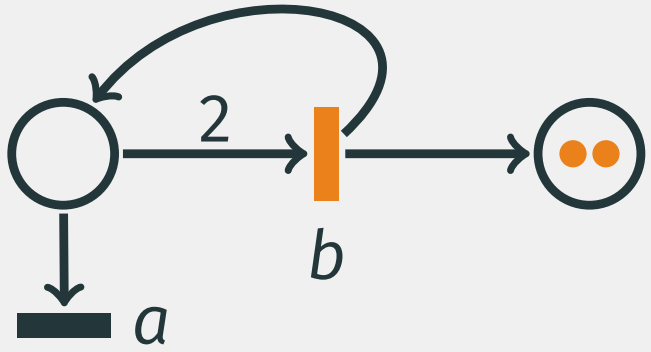
Coverability in continuous Petri nets



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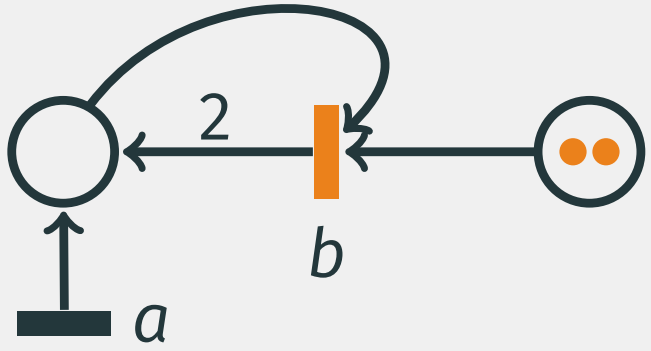
Coverability in continuous Petri nets



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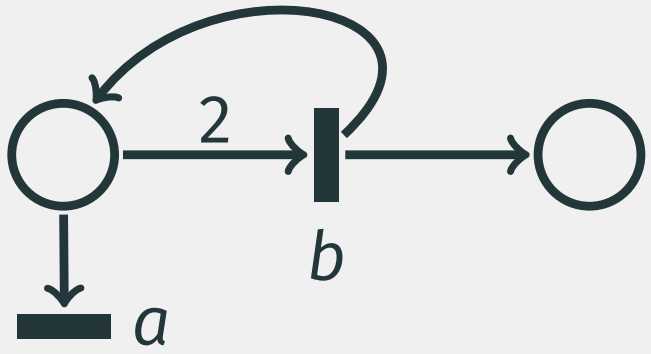
Coverability in continuous Petri nets



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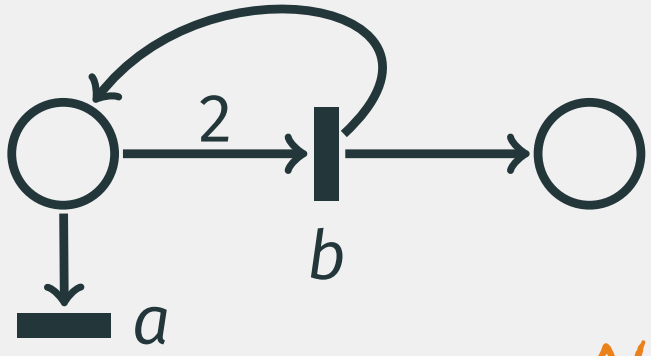
Coverability in continuous Petri nets



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Not $\mathbb{Q}$-coverable from

$\boldsymbol{m}$ is $\mathbb{Q}$-coverable from $\boldsymbol{m}_0$ iff... **Fraca & Haddad '13**

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Coverability in continuous Petri nets

Polynomial time!

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Coverability in continuous Petri nets

Logical characterization — **Contribution**

$\mathbb{Q}$-coverability can be encoded in a linear size formula of

existential FO$(\mathbb{Q}_{\geq 0}, +, <)$

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Coverability in continuous Petri nets

Logical characterization — **Contribution**

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$$\text{existential FO}(\mathbb{N}, \quad +, <)$$

Even better approximation

$\boldsymbol{m}$ is $\mathbb{Q}$-coverable from $\boldsymbol{m}_0$ iff... — **Fraca & Haddad '13**

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Coverability in continuous Petri nets

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Coverability in continuous Petri nets

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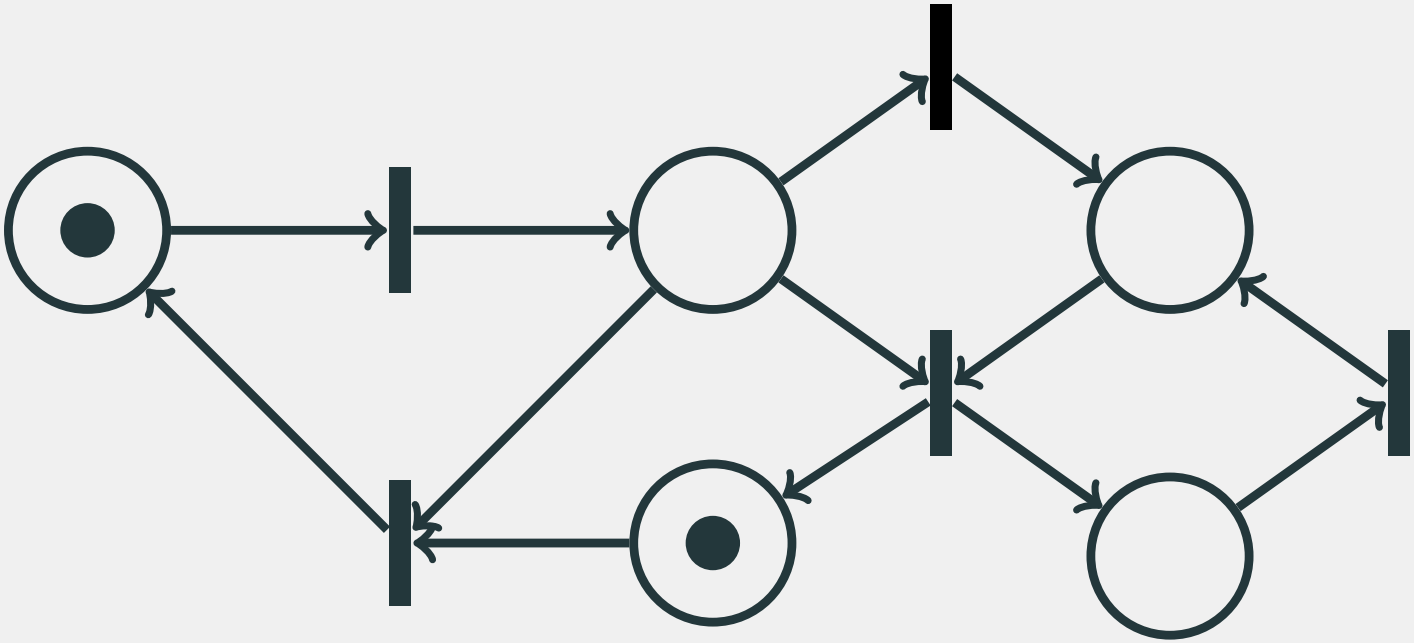
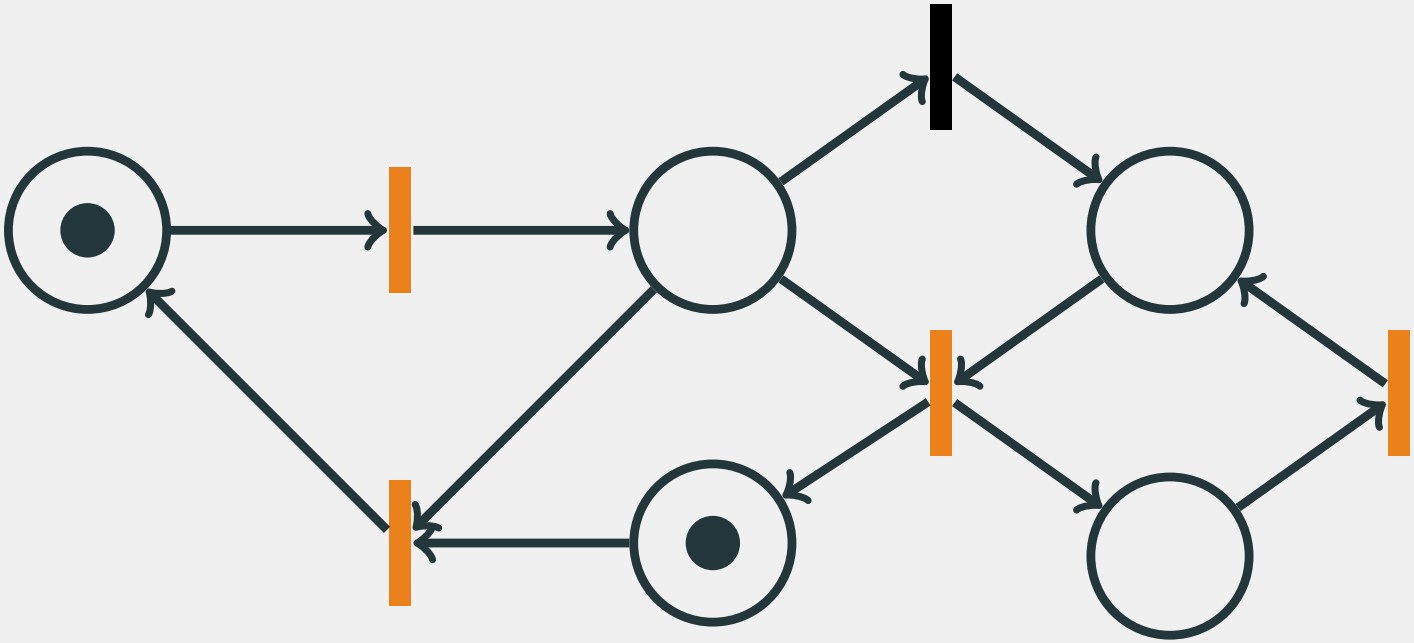
More subtle

Encoding the firing set conditions



Testing whether some transitions can be fired from initial marking

Encoding the firing set conditions



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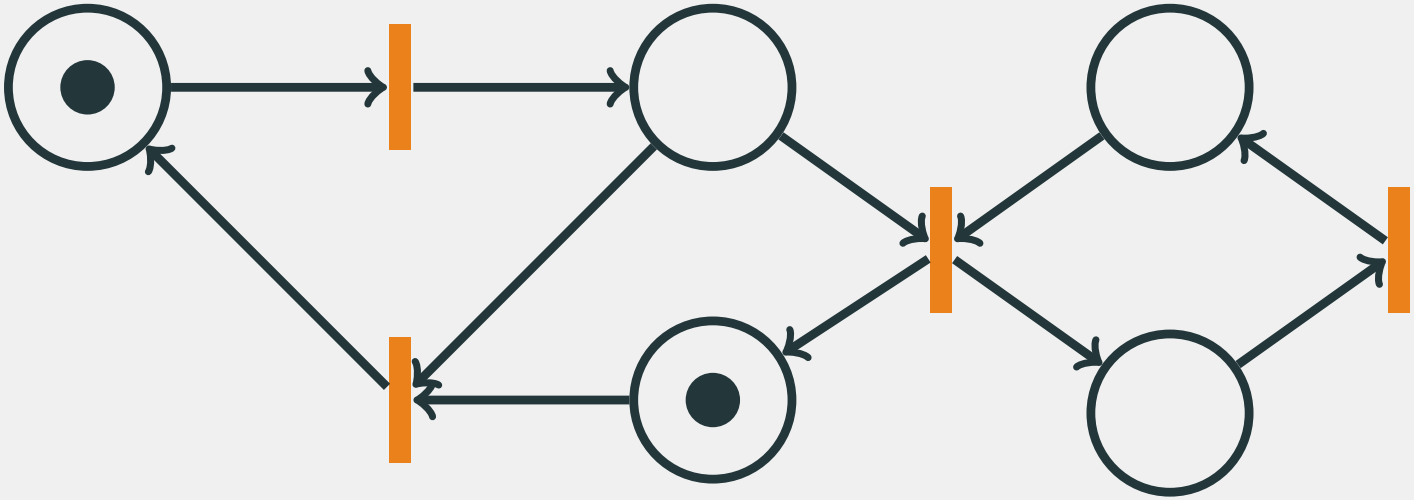

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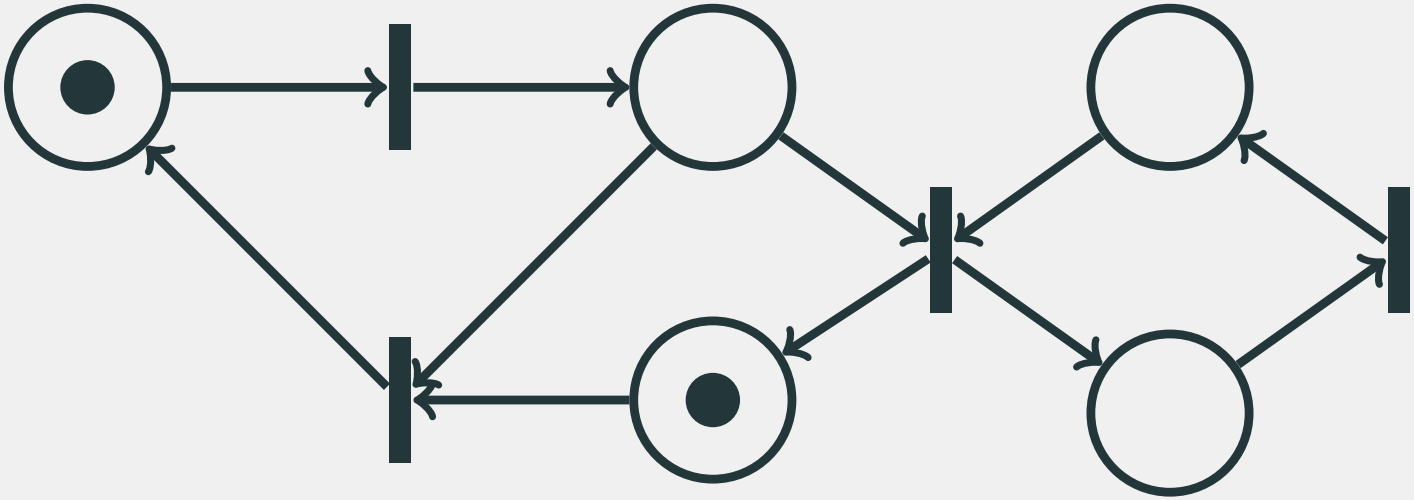
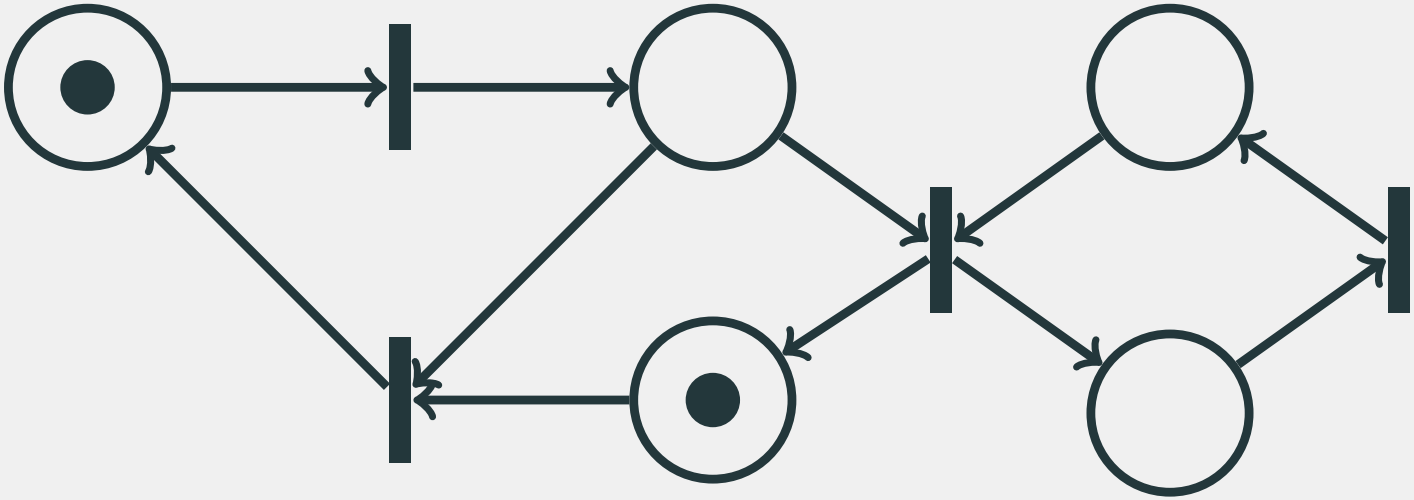


Simulate a "breadth-first" transitions firing

Encoding the firing set conditions

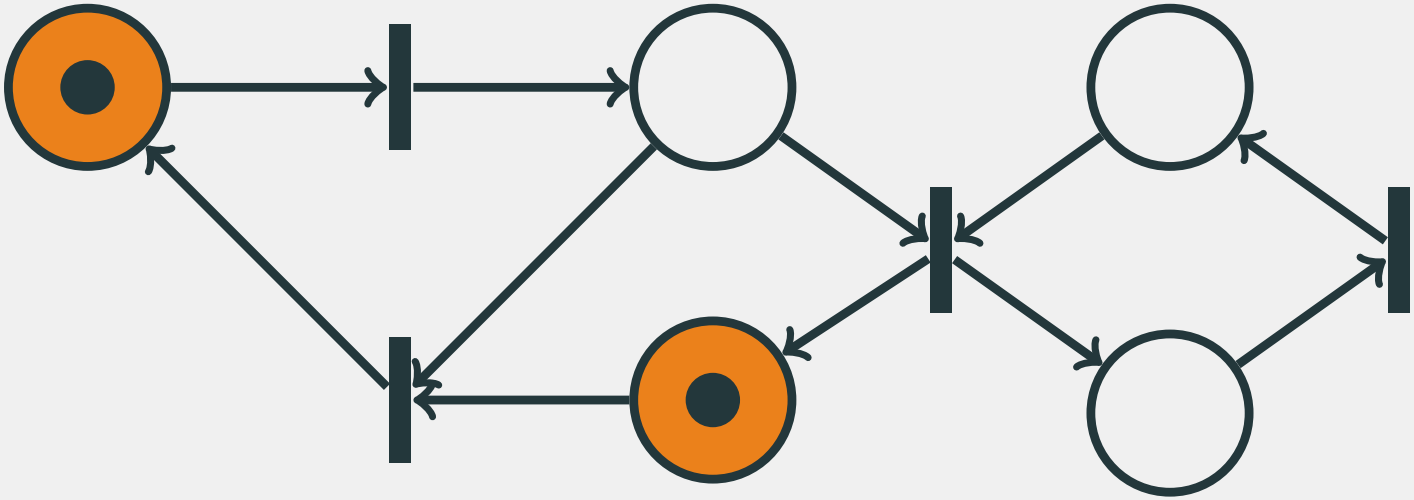


Simulate a "breadth-first" transitions firing

by numbering places/transitions

(Verma, Seidl & Schwentick '05)

Encoding the firing set conditions



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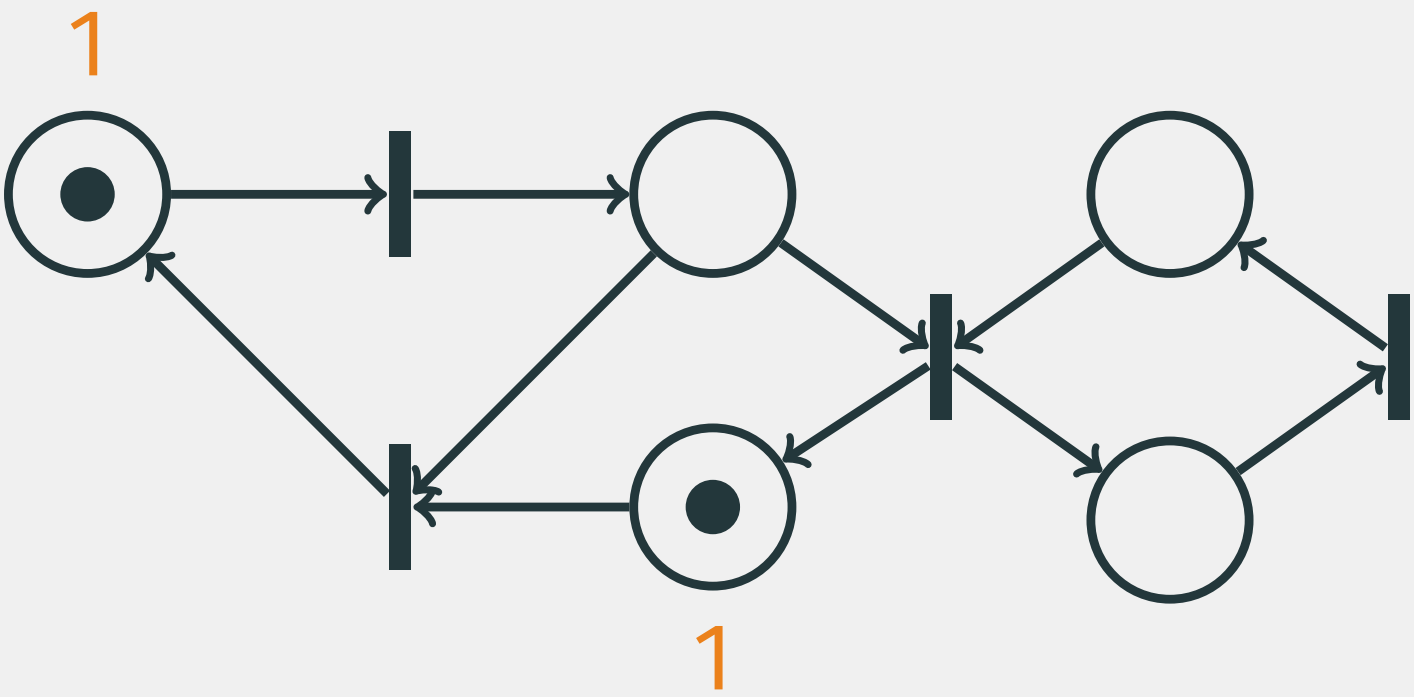

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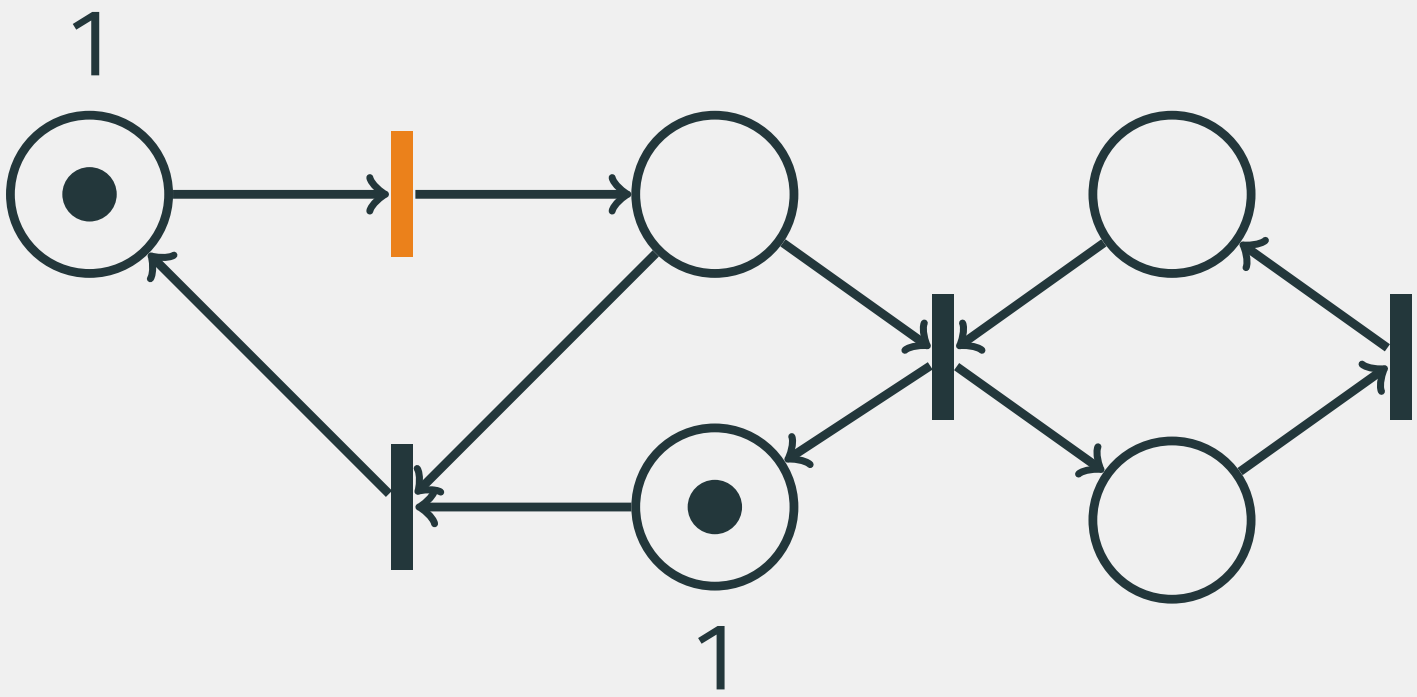

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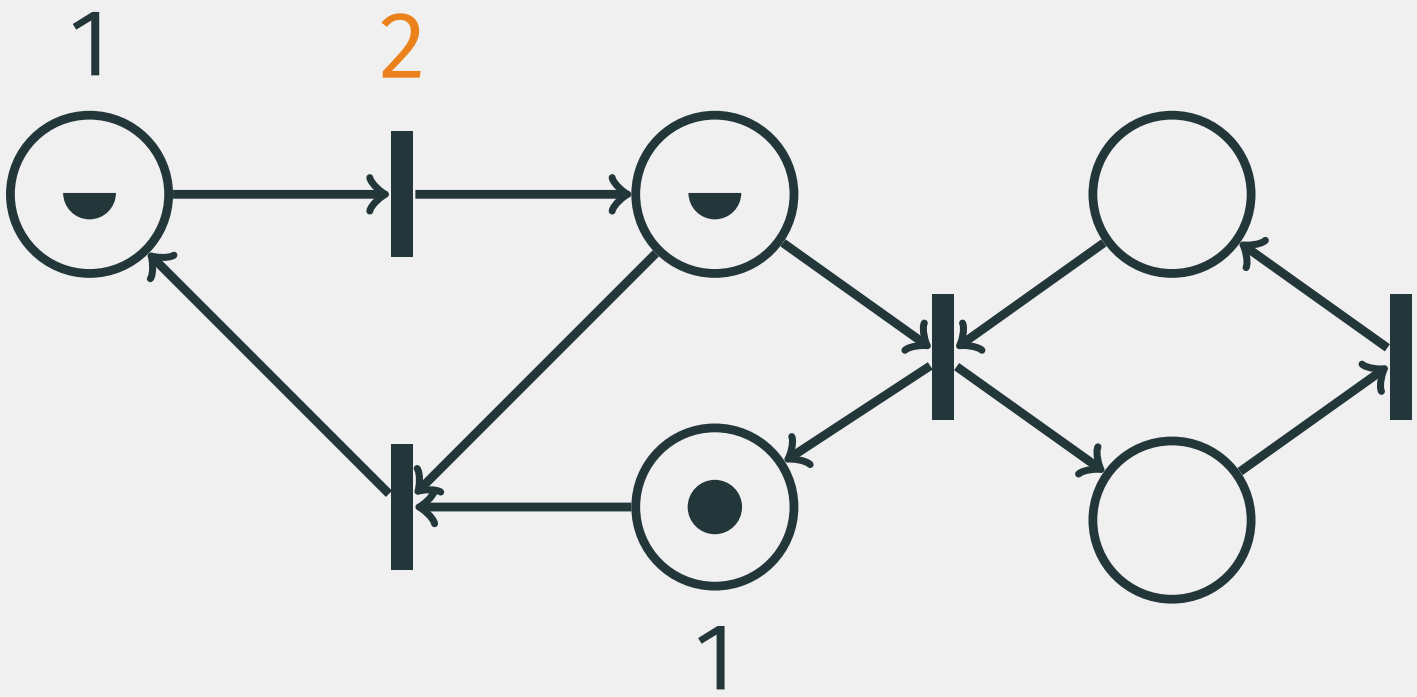

Simulate a "breadth-first" transitions firing by numbering places/transitions

(Verma, Seidl & Schwentick '05)

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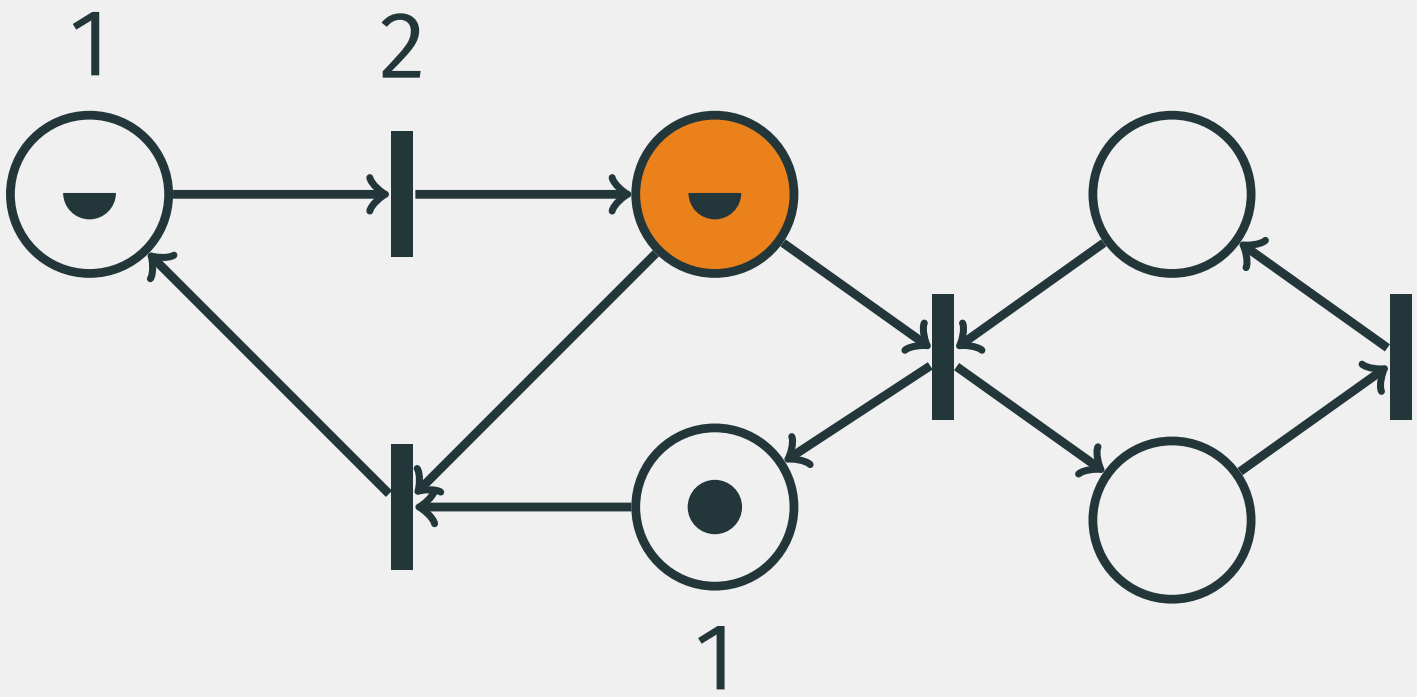

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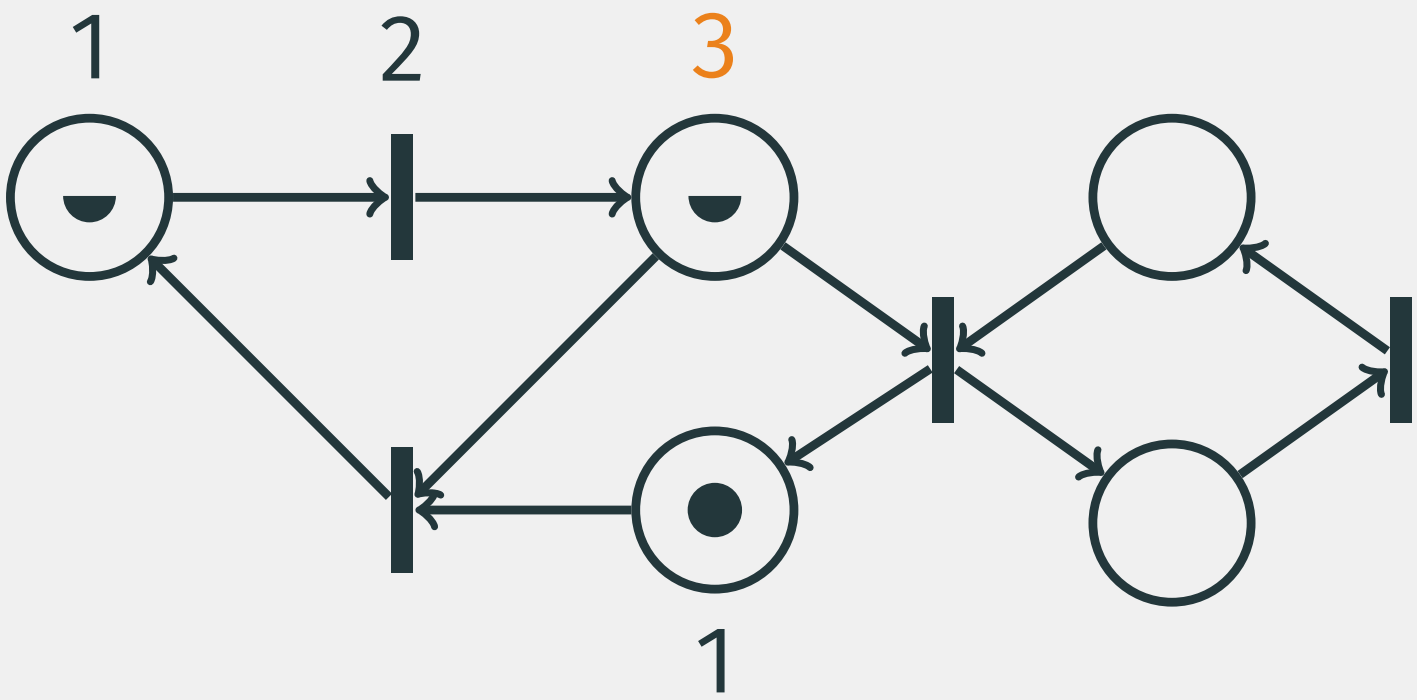

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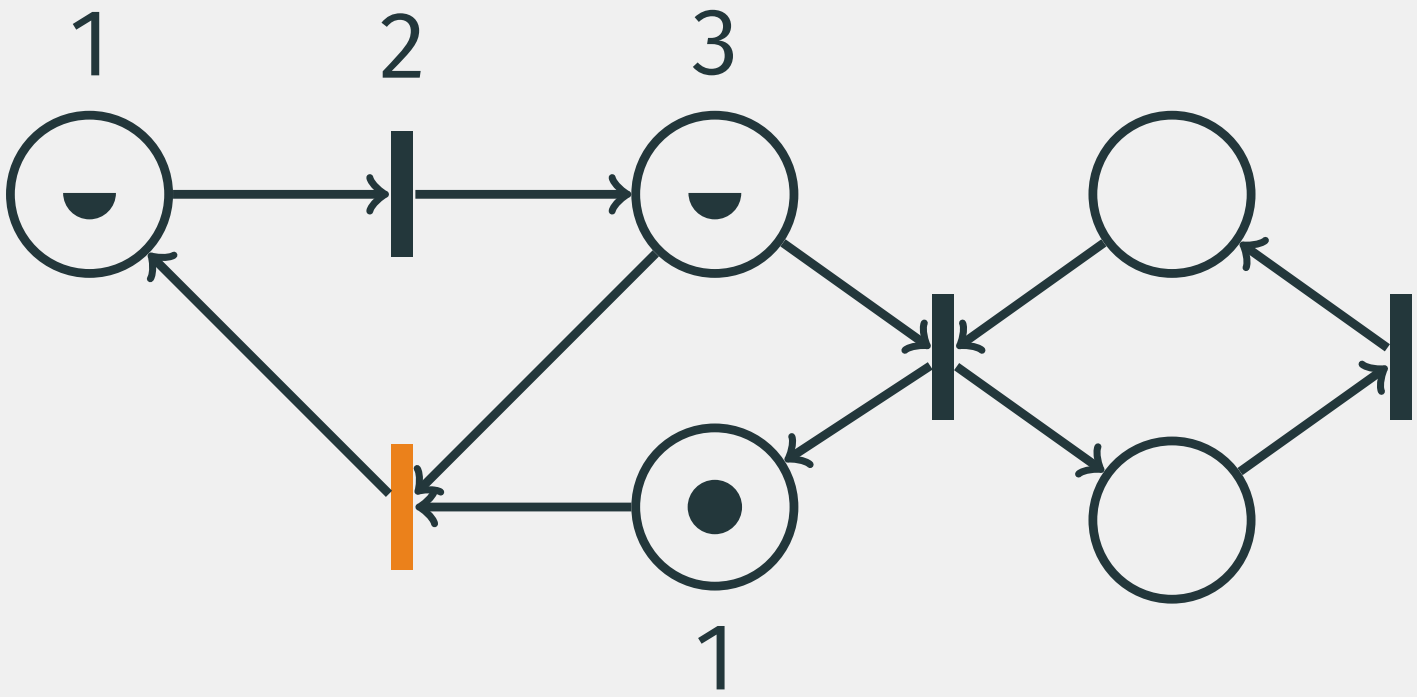

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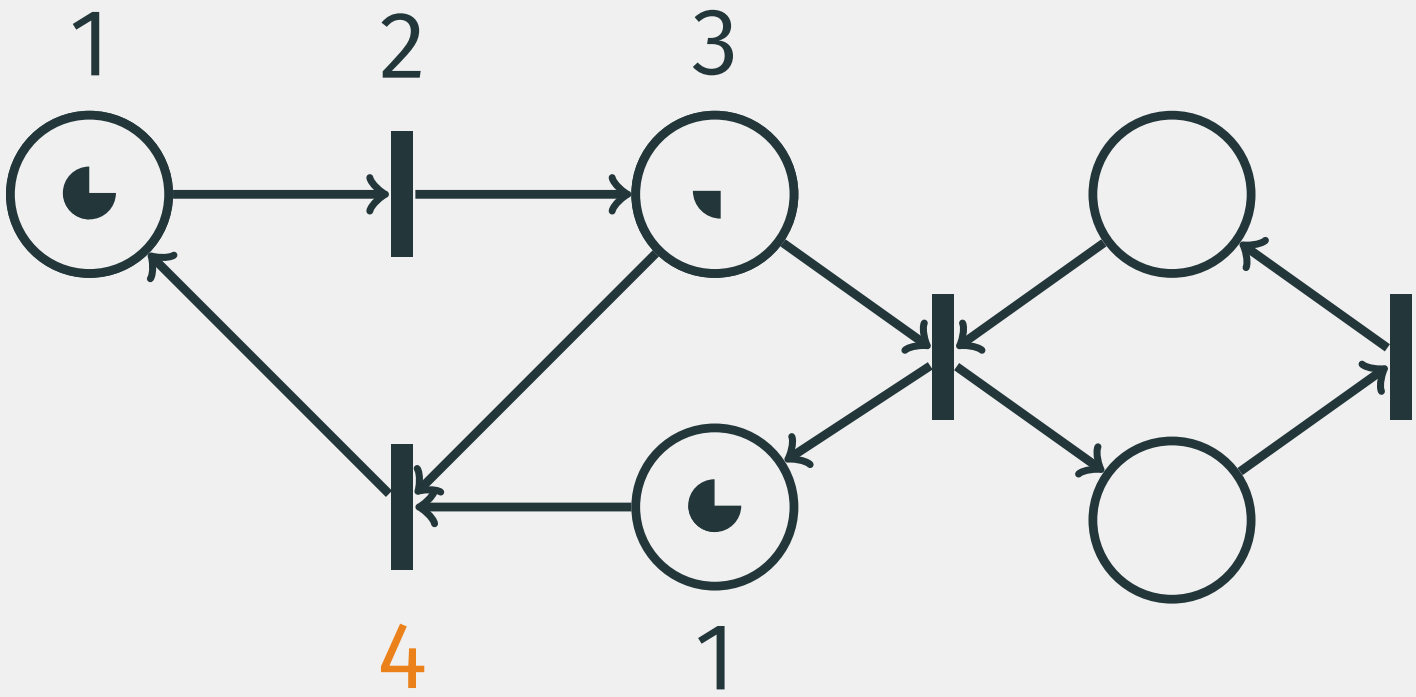

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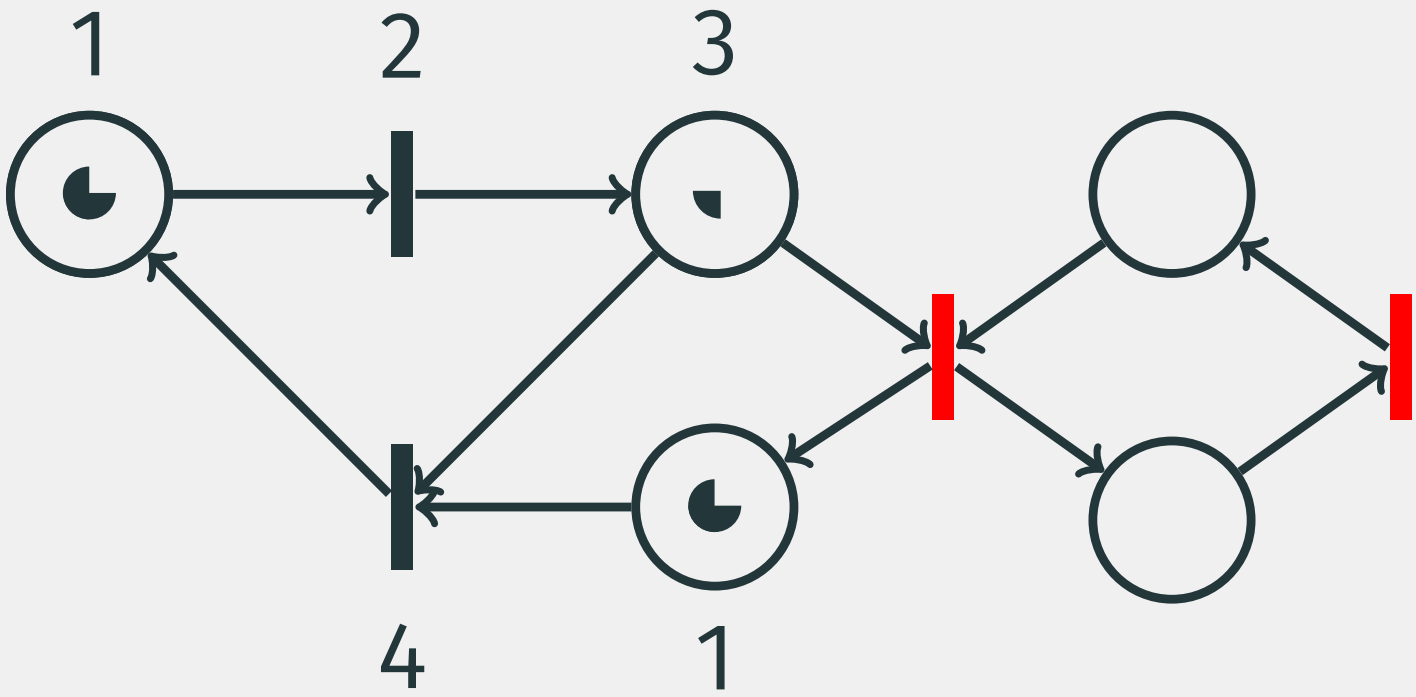

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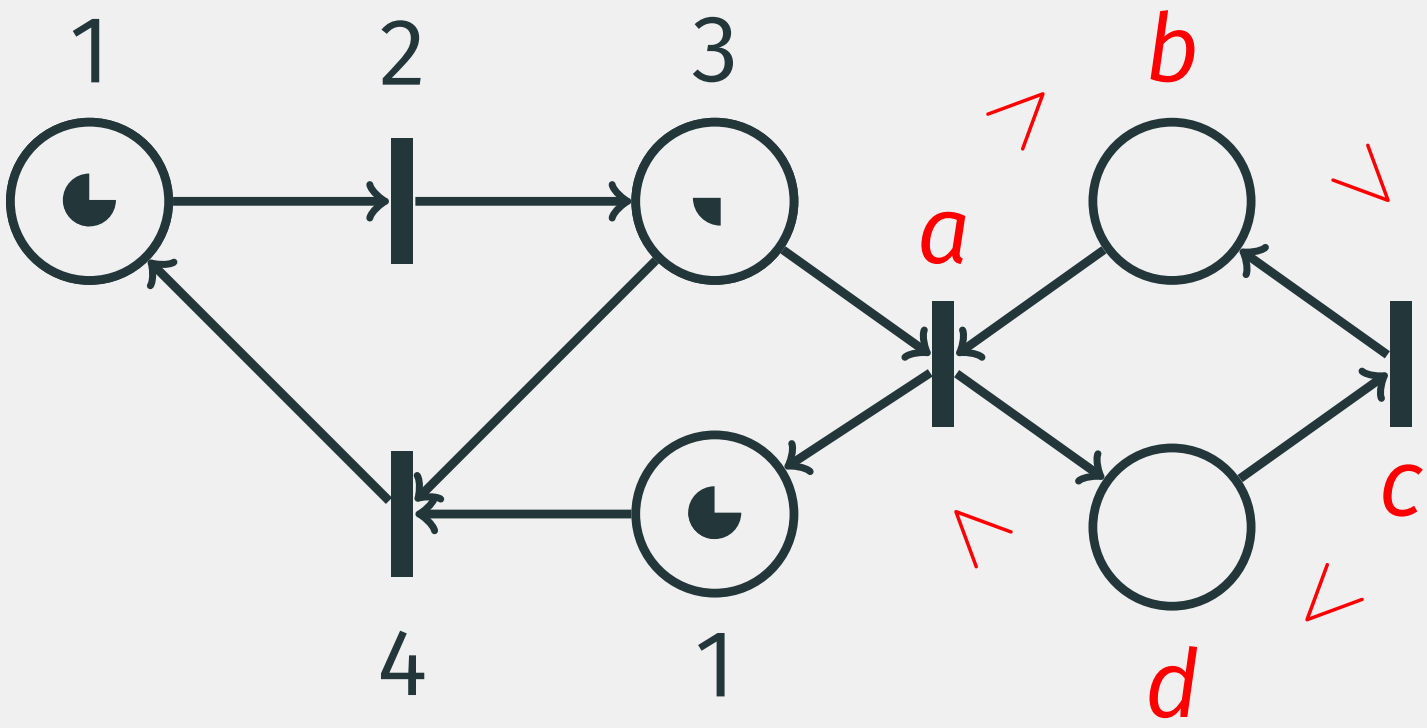

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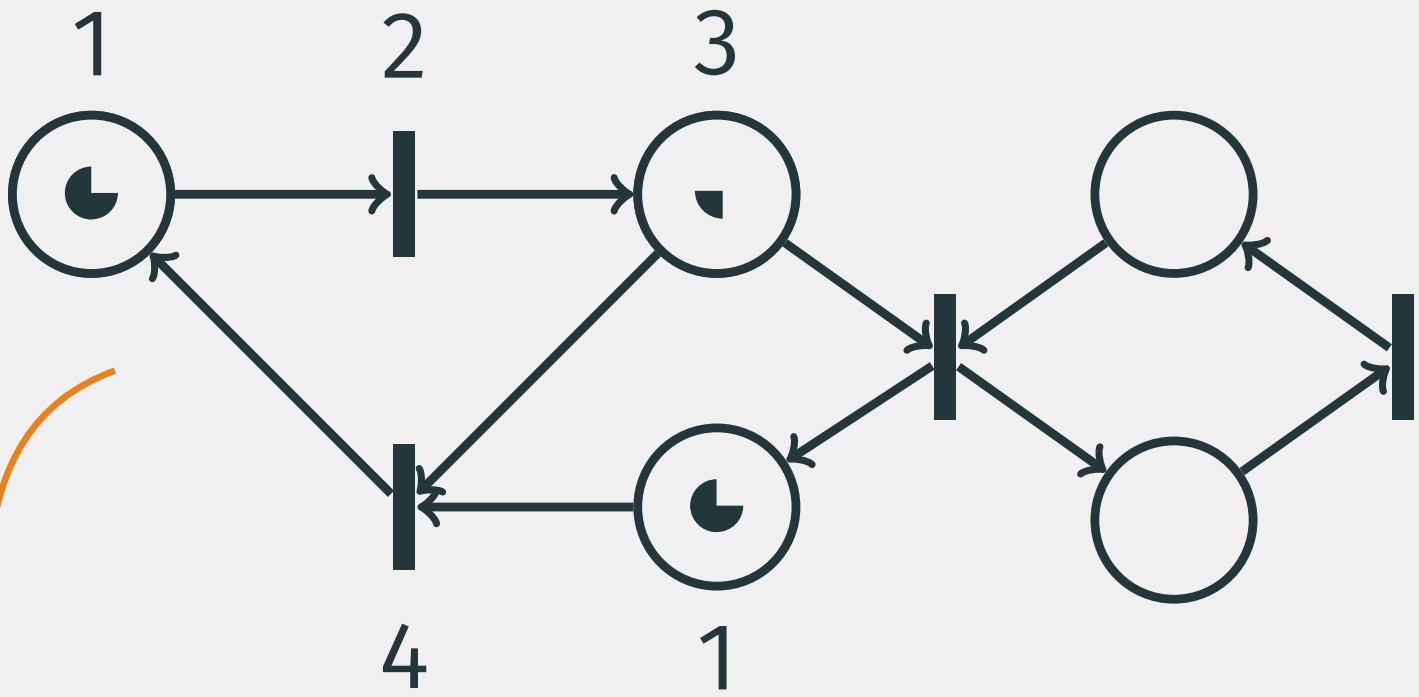

$$\varphi(\boldsymbol{x}) = \exists \boldsymbol{y} : \bigwedge_{p \in P} \boldsymbol{y}(p) > 0 \rightarrow \bigwedge_{t \in {}^\bullet p} \boldsymbol{y}(t) < \boldsymbol{y}(p) \cdots$$

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if target marking m is not Q-coverable:
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SMT solver guidance

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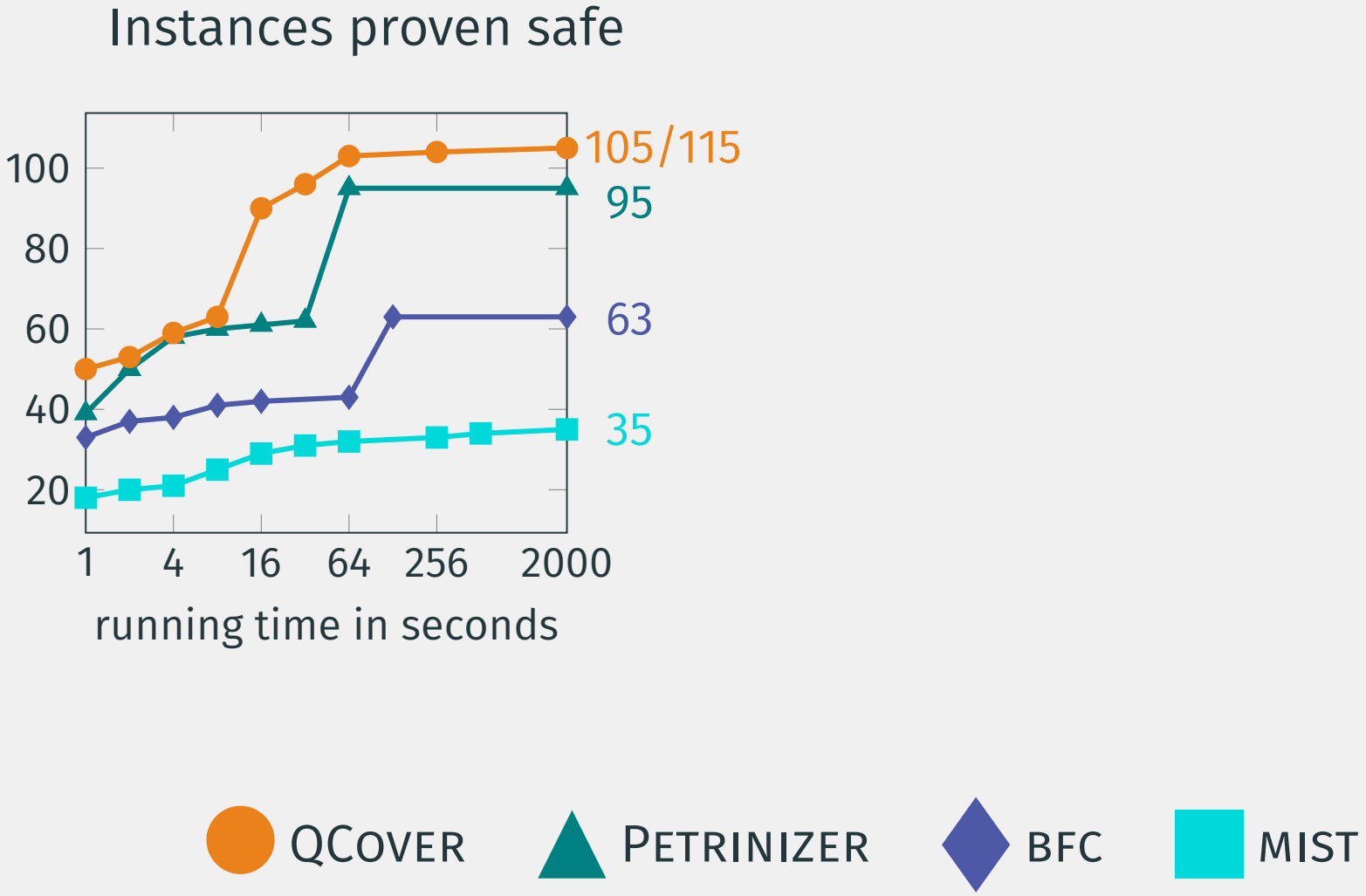
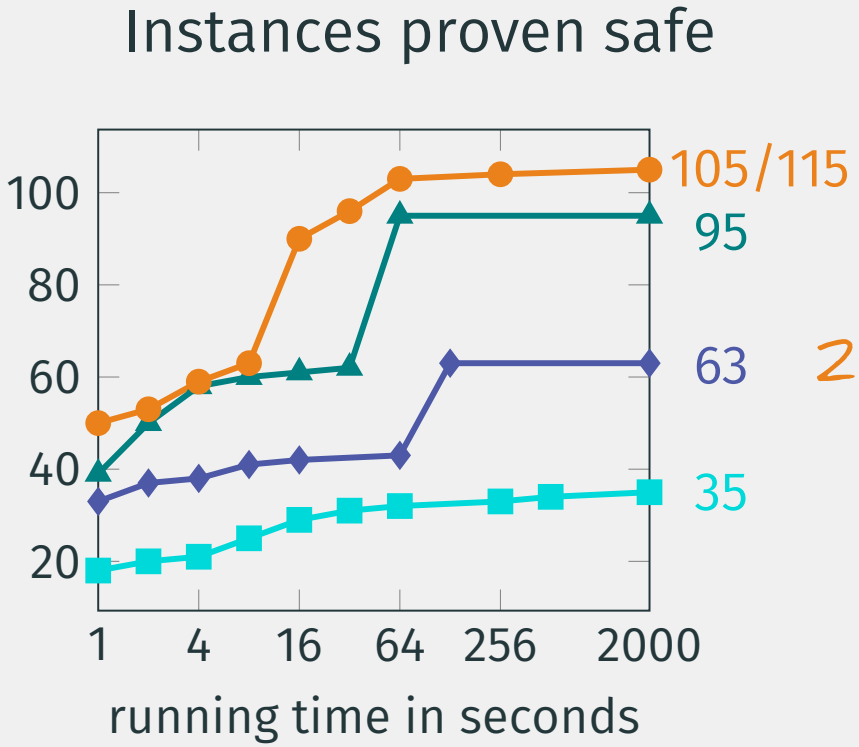

Instances proven safe
100
80
60
40
20
1
4
16
64
256
2000
running time in seconds
105/115
95
63
35
QCover
Petrinizer
BFC
MIST

Benchmarks



Largest nets proved safe:

21143 places
7150 trans.
42 secs.

6690 places
11934 trans.
21 secs.

754 places
27370 trans.
3 secs.

QCover Petrinizer BFC Mist

Benchmarks

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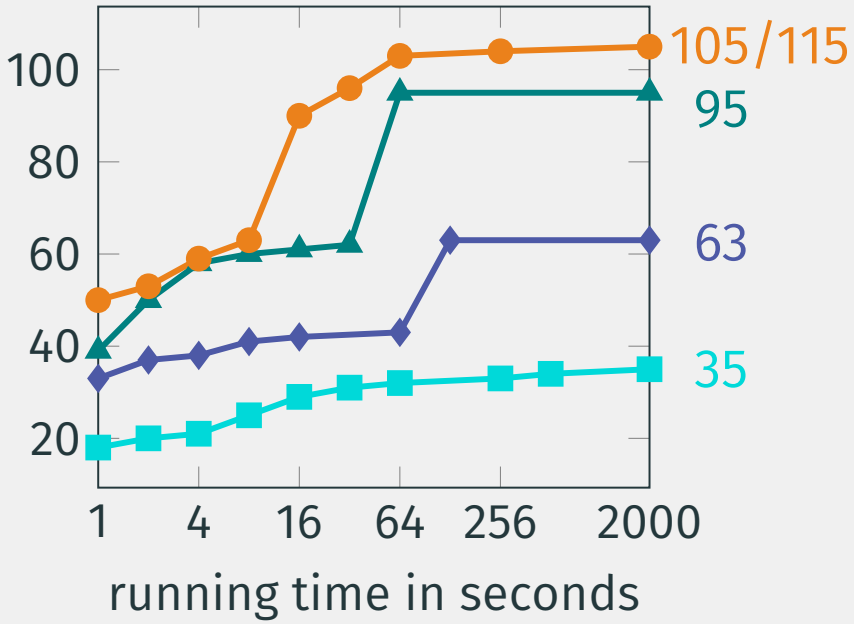
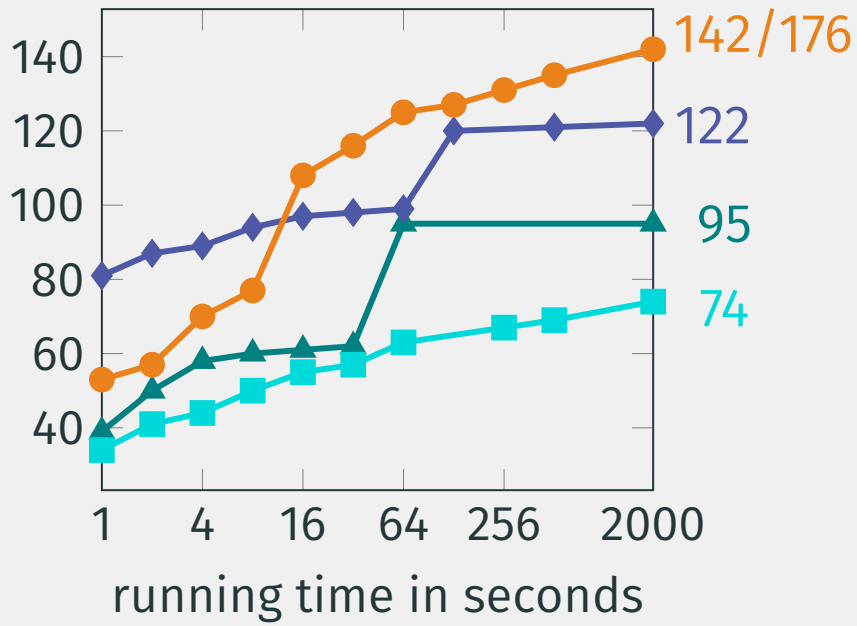


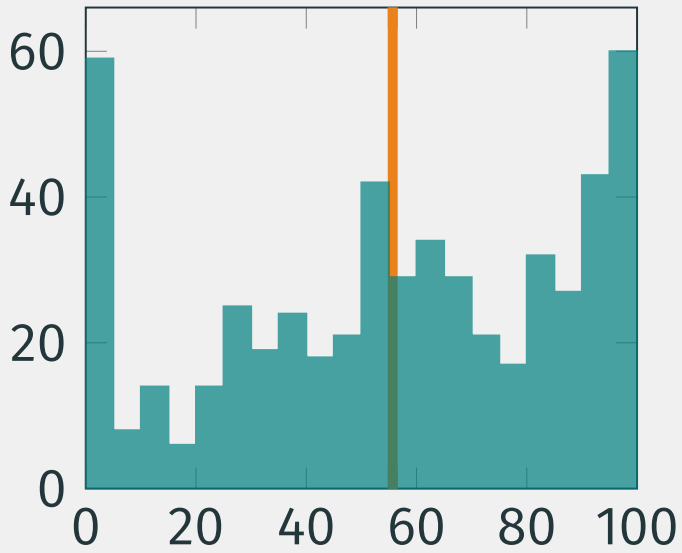
Instances proven safe or unsafe

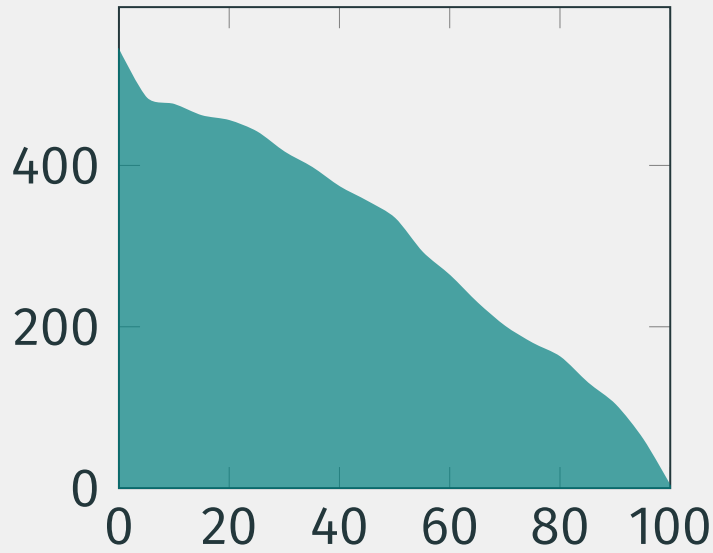


QCover Petrinizer BFC Mist

Benchmarks

Markings pruning efficiency across all iterations





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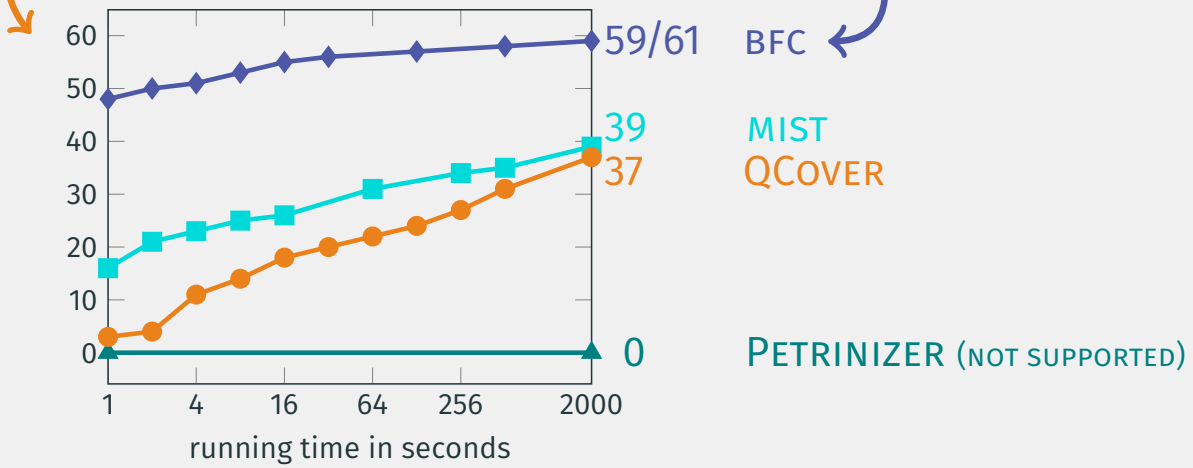

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- Extend to Petri nets with transfer/reset arcs

Thank you! Dank u!