Handling Infinitely Branching WSTS

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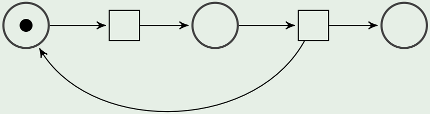
June 16, 2014

Well-structured transition systems (WSTS) encompass a large number of infinite state systems.

Example of WSTS: Petri nets



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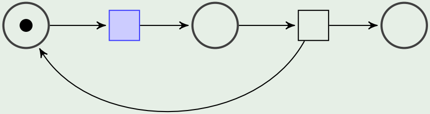

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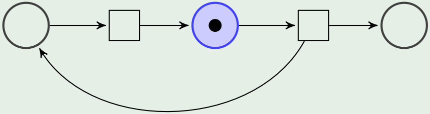

Multiple decidability results are known for finitely branching WSTS.

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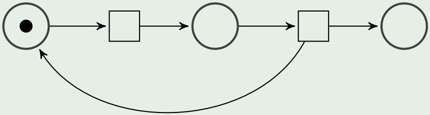
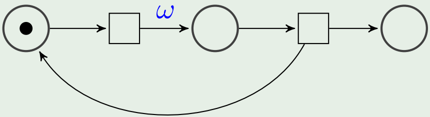


$$\mathrm{Post}(\odot \; \bigcirc \; \bigcirc) \;=\; \bigcirc \; \odot \; \bigcirc$$

How to handle infinitely branching WSTS such as systems with infinitely many initial states, and parametric systems?

Example of WSTS: ω–Petri nets



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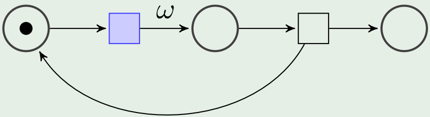

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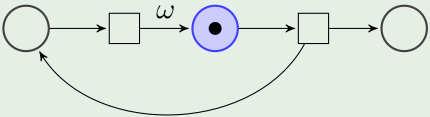

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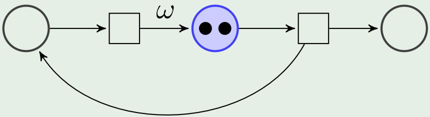

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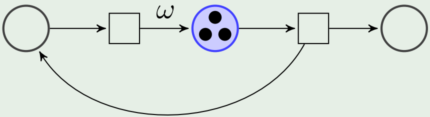

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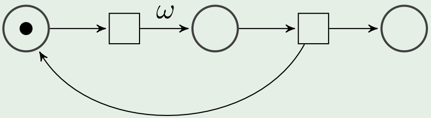
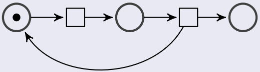


Well-structured transition system (Finkel ICALP'87, Finkel & Schnoebelen TCS'01)

$S = (X, \to, \leq)$ where

- X set,
- $\to \subseteq X \times X$,
- monotony,
- well-quasi-ordered.



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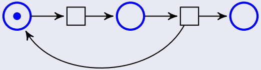

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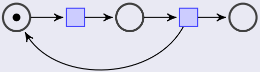

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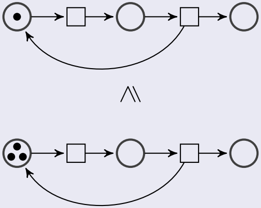

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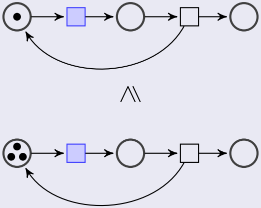

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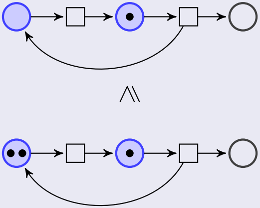

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- monotony,
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  $\forall x_0, x_1, \ldots \ \exists i < j$ s.t. $x_i \leq x_j$.

Branching

A WSTS $(X, \to, \leq)$ is *finitely branching* if $\mathrm{Post}(x)$ is finite for every $x \in X$.

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- Much more.

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- Parametric WSTS.

Objective

We want to study the usual reachability problems for these infinitely branching systems, e.g.,

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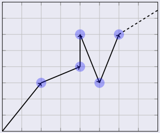
- Termination,



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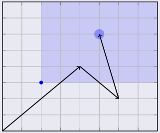
- Termination,
- Coverability,



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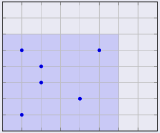
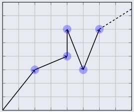
- Termination,
- Coverability,
- Boundedness.



Termination

Input: $(X, \to, \leq)$ a WSTS, $x_0 \in X$.

Question: $\nexists x_0 \to x_1 \to x_2 \to \ldots$?



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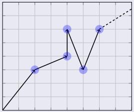

Theorem (Finkel & Schnoebelen TCS'01)

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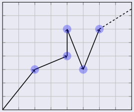

Theorem (deduced from Dufourd, Jančar & Schnoebelen ICALP'99)

Termination is undecidable for infinitely branching WSTS.

Strong termination

Input: $(X, \rightarrow, \leq)$ a WSTS, $x_0 \in X$.

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Remark

Strong termination and termination are the same in finitely branching WSTS.

Strong termination

Input: $(X, \rightarrow, \leq)$ a WSTS, $x_0 \in X$.

Question: $\exists k$ bounding length of executions from x_0?

Theorem (B., Finkel & McKenzie ICALP'14)

Strong termination is decidable for infinitely branching WSTS under some assumptions.

Issues with finite branching techniques

Some techniques for WSTS based on finite reachability trees; impossible for infinite branching.

Some rely on upward closed sets; what about downward closed, in particular with infinite branching?

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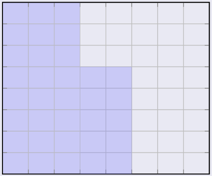
A tool

Develop from the WSTS *completion* introduced by Finkel & Goubault-Larrecq 2009.

Ideals

$I \subseteq X$ is an *ideal* if

- downward closed: $I = \downarrow I$,



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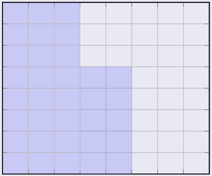

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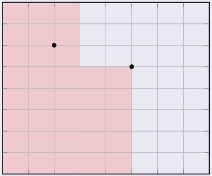

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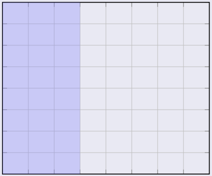
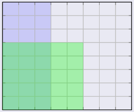


Theorem (Finkel & Goubault-Larrecq ICALP'09; Goubault-Larrecq '14)

$$D \text{ downward closed} \implies D = \bigcup_{\text{finite}} \text{Ideals}$$



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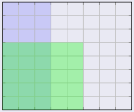

Corollary (B., Finkel & McKenzie ICALP'14)

Every downward closed set decomposes canonically as the union of its maximal ideals.

Completion (B., Finkel & McKenzie ICALP'14)

The *completion* of $S = (X, \rightarrow_S, \leq)$ is $\widehat{S} = (\widehat{X}, \rightarrow_{\widehat{S}}, \subseteq)$ such that

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- $\widehat{X} = \text{Ideals}(X)$,
- $I \to_{\widehat{S}} J$ if $\downarrow\text{Post}(I) = \underbrace{\ldots \cup J \cup \ldots}_{\text{canonical decomposition}}$

Theorem (B., Finkel & McKenzie ICALP'14)

Let $S = (X, \rightarrow_S, \leq)$ be a WSTS, then $\widehat{S} = (\widehat{X}, \rightarrow_{\widehat{S}}, \subseteq)$ such that

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- $\widehat{S}$ is finitely branching,
- $\widehat{S}$ has (strong) monotony,
- $\widehat{S}$ is *not always* a WSTS (Jančar IPL'99).

Relating executions of S and $\widehat{S}$

Let $S = (X, \rightarrow_S, \leq)$ be a WSTS, then

- if $x \xrightarrow{k}_S y$,

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Relating executions of S and $\widehat{S}$

Let $S = (X, \rightarrow_S, \leq)$ be a WSTS with transitive monotony, then

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Relating executions of S and $\widehat{S}$

Let $S = (X, \rightarrow_S, \leq)$ be a WSTS with strong monotony, then

- if $x \xrightarrow{k}_S y$, then for every ideal $I \supseteq {\downarrow} x$ there exists an ideal $J \supseteq {\downarrow} y$ such that $I \xrightarrow{k}_{\widehat{S}} J$,
- if $I \xrightarrow{k}_{\widehat{S}} J$, then for every $y \in J$ there exists $x \in I$ such that $x \xrightarrow{k}_S y' \geq y$.

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Post-effectiveness

Possible to compute cardinality of

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Proof

- Executions bounded in S iff bounded in $\widehat{S}$.

Theorem (B., Finkel & McKenzie ICALP'14)

Strong termination is decidable for infinitely branching WSTS with transitive monotony and such that $\widehat{S}$ is a post-effective WSTS.

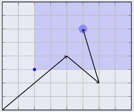
Proof

- Executions bounded in S iff bounded in $\widehat{S}$.
- $\widehat{S}$ finitely branching, can decide termination in $\widehat{S}$ by Finkel & Schnoebelen 2001.

Coverability

Input: $(X, \rightarrow, \leq)$ a WSTS, $x_0, x \in X$.

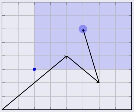
Question: $x_0 \xrightarrow{*} x' \geq x$?



Coverability

Input: $(X, \to, \leq)$ a WSTS, $x_0, x \in X$.

Question: $x \in \downarrow \mathrm{Post}^*(x_0)$?



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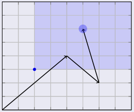

Forward method

Coverability:

- Enumerate executions $\downarrow x_0 \xrightarrow{*}_{\widehat{S}} I$,
- Accept if $x \in I$.

Coverability

Input: $(X, \rightarrow, \leq)$ a WSTS, $x_0, x \in X$.

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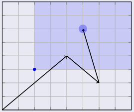


Forward method

Coverability:

- Enumerate executions ${\downarrow}x_0 \xrightarrow{*}_{\widehat{S}} I$,
- Accept if $x \in I$.

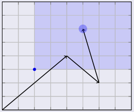
Non coverability:

- Enumerate $D \subseteq X$ downward closed, $x_0 \in D$ and ${\downarrow}\mathrm{Post}_S(D) \subseteq D$
- Reject if $x \notin D$.

Coverability

Input: $(X, \rightarrow, \leq)$ a WSTS, $x_0, x \in X$.

Question: $x \in \downarrow \mathrm{Post}^*(x_0)$?



Forward method

Coverability:

- Enumerate executions $\downarrow x_0 \xrightarrow{*}_{\widehat{S}} I$,
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Non coverability:

- Enumerate $D = I_1 \cup \ldots \cup I_k$
- Reject if $x \notin D$.

Coverability

Input: $(X, \to, \leq)$ a WSTS, $x_0, x \in X$.

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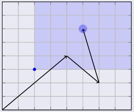

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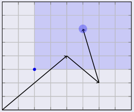

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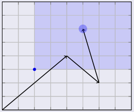

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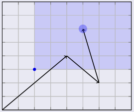

Forward method

Coverability:

- Enumerate executions $\downarrow x_0 \xrightarrow{*}_{\widehat{S}} I$,
- Accept if $x \in I$.

Non coverability:

- Enumerate $D \subseteq X$ downward closed, $\exists j$ s.t. $\downarrow x_0 \subseteq I_j$
- Reject if $x \notin D$.

Coverability

Input: $(X, \to, \leq)$ a WSTS, $x_0, x \in X$.

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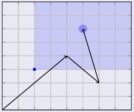

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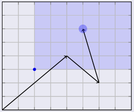
Coverability

Input: $(X, \rightarrow, \leq)$ a WSTS, $x_0, x \in X$.

Question: $x_0 \in \uparrow \mathrm{Pre}^*(\uparrow x)$?



Backward method (Abdulla, Cerans, Jonsson & Tsay IC'00)

Compute $\uparrow \mathrm{Pre}^*(\uparrow x)$ iteratively assuming $\uparrow \mathrm{Pre}(U)$ computable.

Coverability

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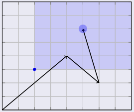

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Further results for infinitely branching WSTS

- Boundedness is decidable for post-effective WSTS with strict monotony,

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- Strong maintainability is decidable for WSTS with strong monotony and such that $\widehat{S}$ is a post-effective WSTS.

Further work

- $\exists$ general class of infinitely branching WSTS with a Karp-Miller procedure?

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- Toward the algorithmics of complete WSTS.

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- Toward the algorithmics of complete WSTS.
- What else can we do with the WSTS completion?

Thank you! Merci!