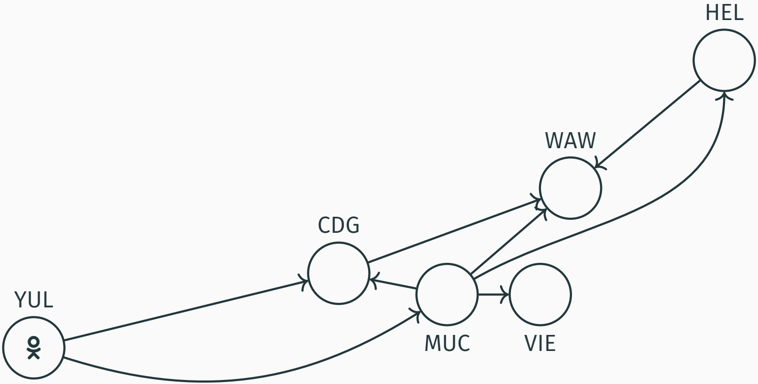
On reachability in subclasses of unordered data Petri nets

Michael Blondin

Université de Sherbrooke

Traveling from Montréal to Warsaw



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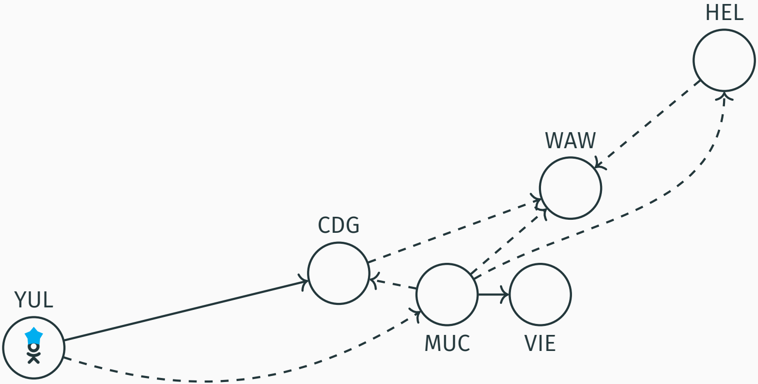

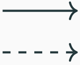

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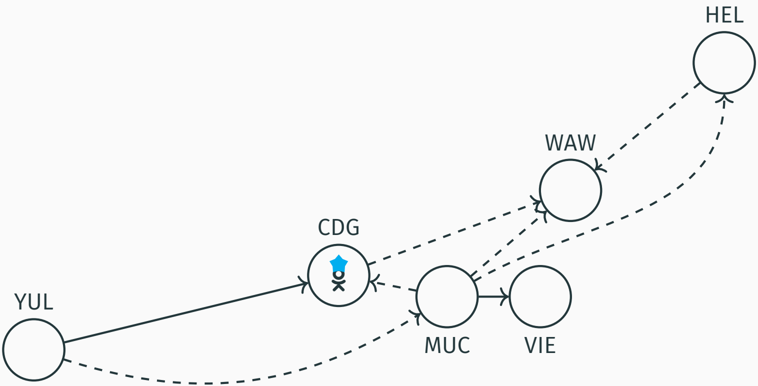

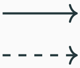

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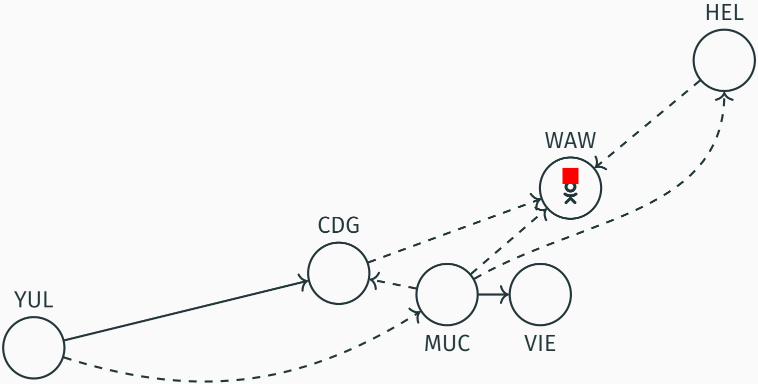

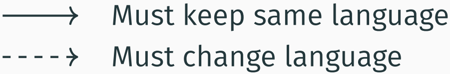

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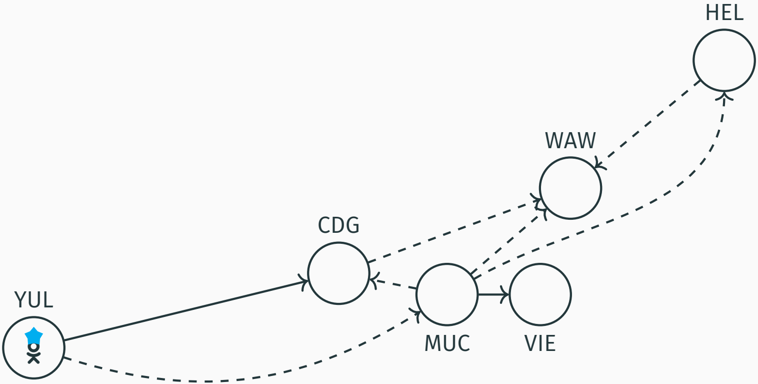

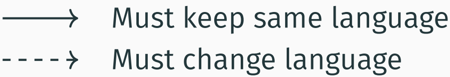

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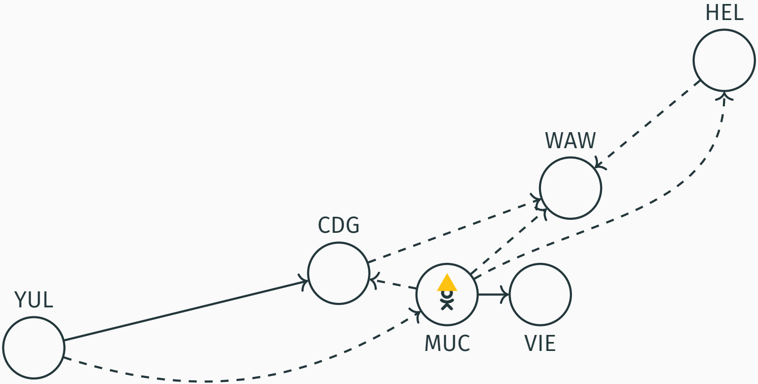

⟶ Must keep same language

- - - -› Must change language

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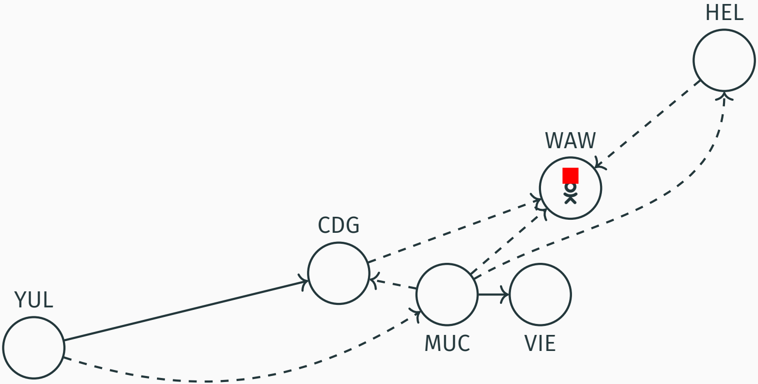

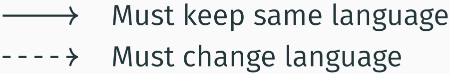

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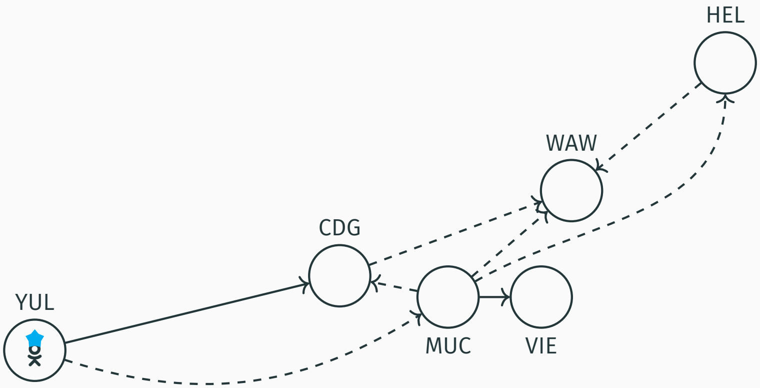

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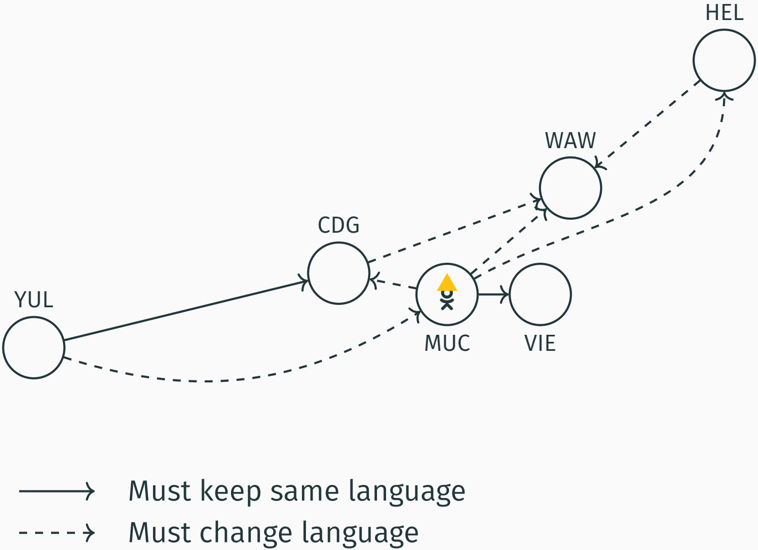

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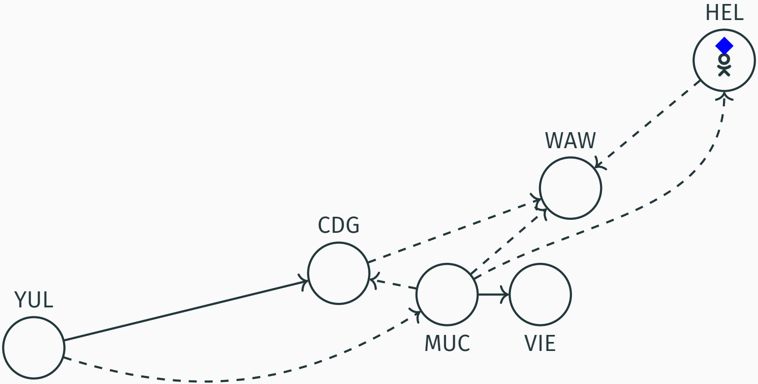

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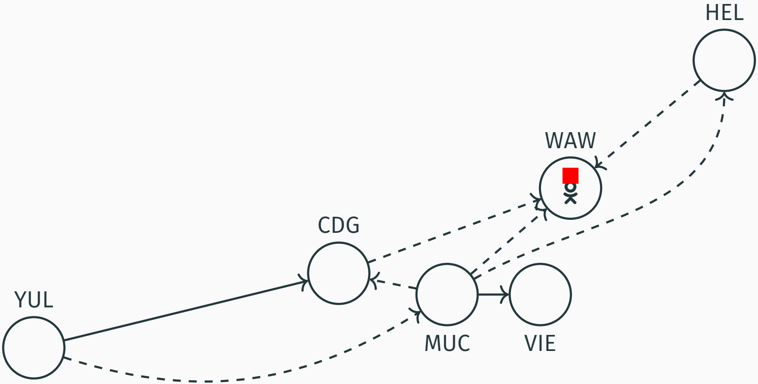

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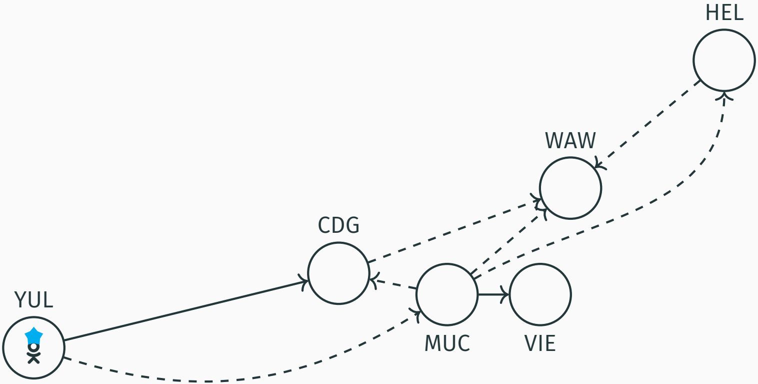

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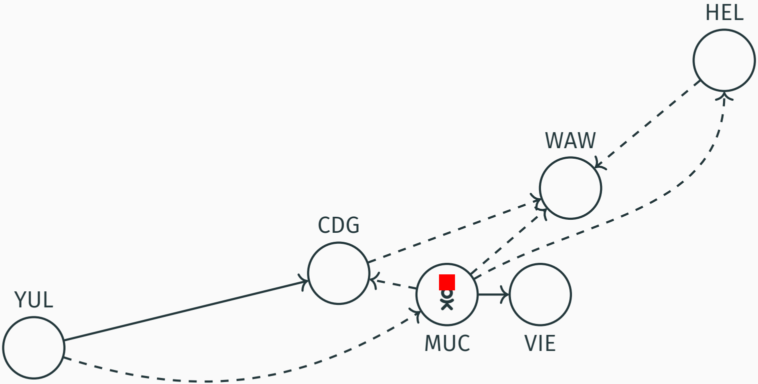

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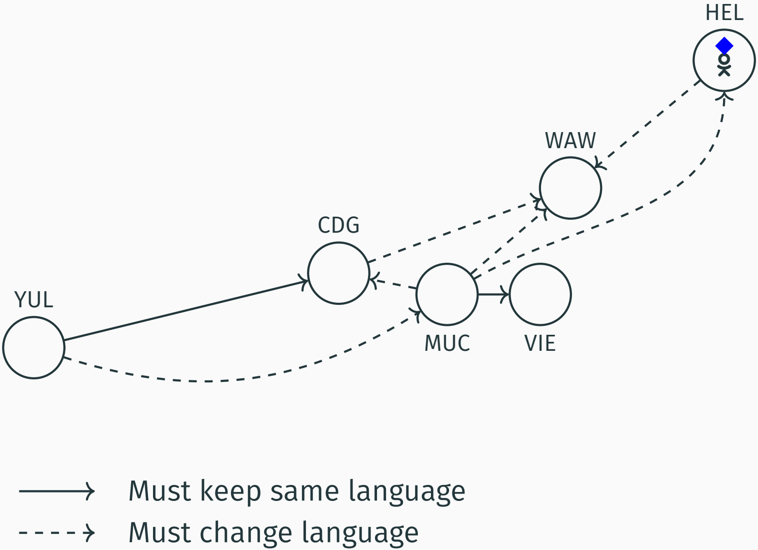

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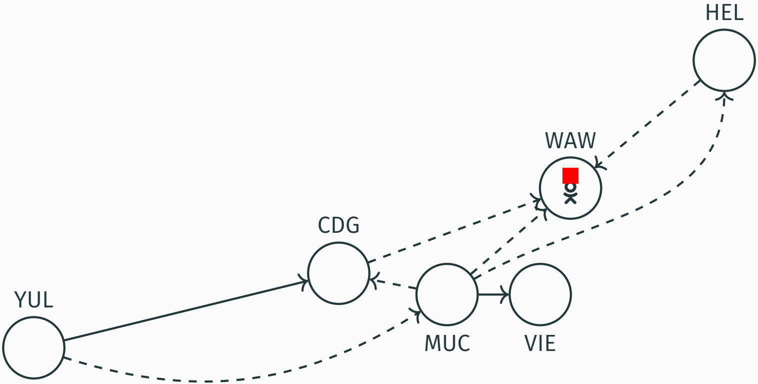

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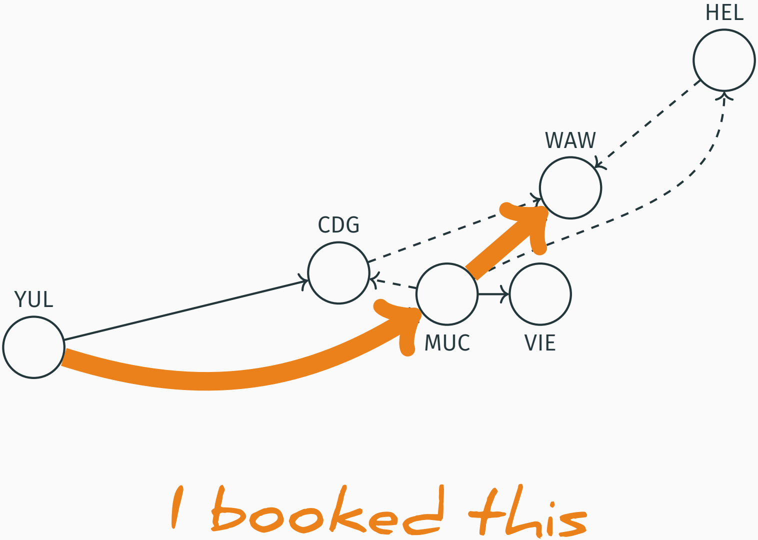

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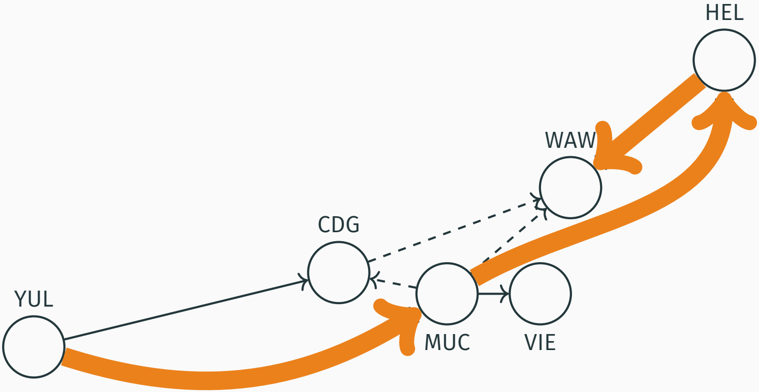
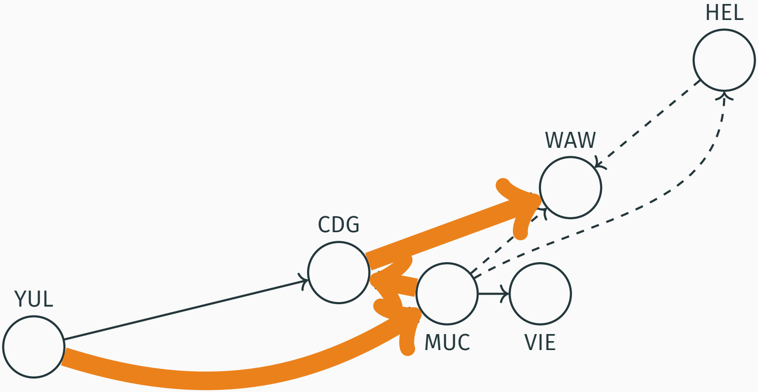


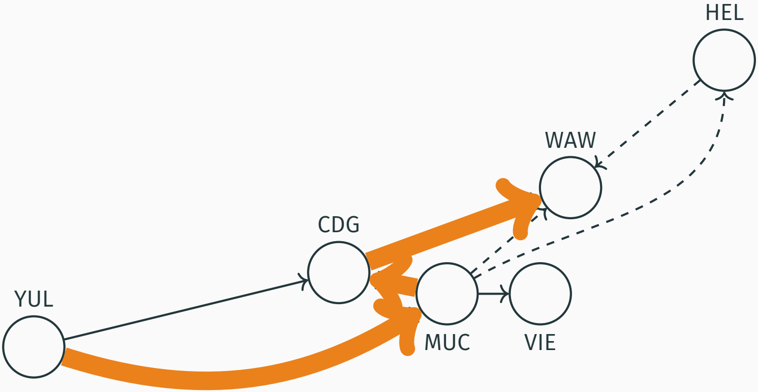
Lufthansa changed it to this!

Traveling from Montréal to Warsaw



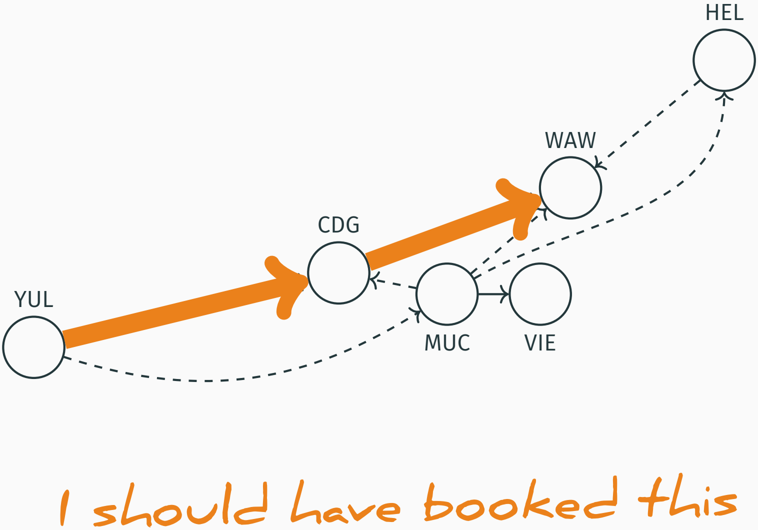
By arguing, I got this...

Traveling from Montréal to Warsaw

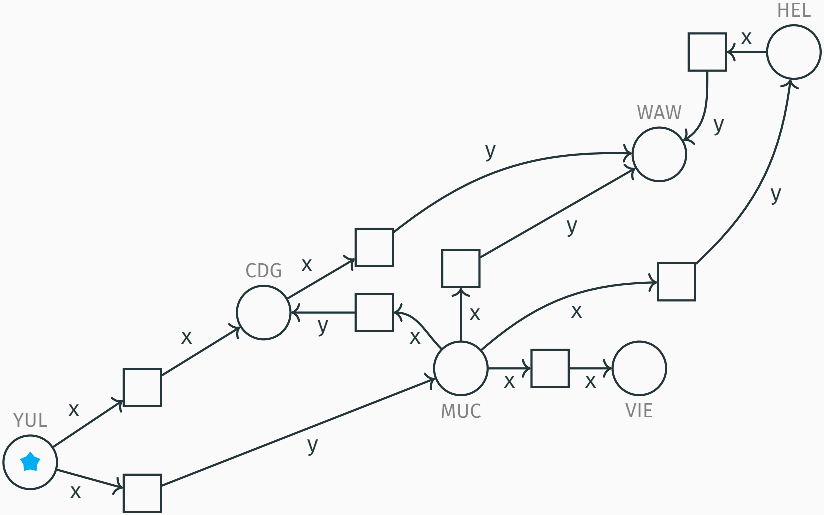


By arguing, I got this...
(travel time: 24 hours ☺)

Traveling from Montréal to Warsaw



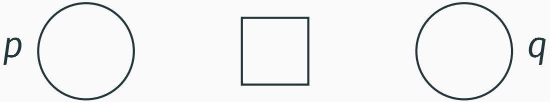
Unordered data Petri nets (UDPNs)



Unordered data Petri nets (UDPNs)

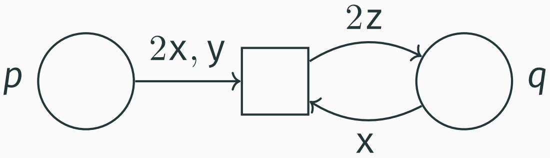
Places (finite set P)

Unordered data Petri nets (UDPNs)



Transitions (finite set T)

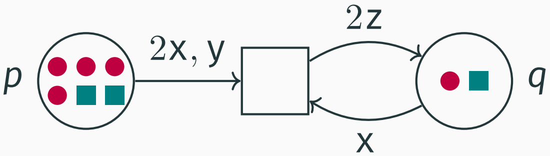
Unordered data Petri nets (UDPNs)



Arcs

(labeled with variables weighted over $\mathbb{N}$)

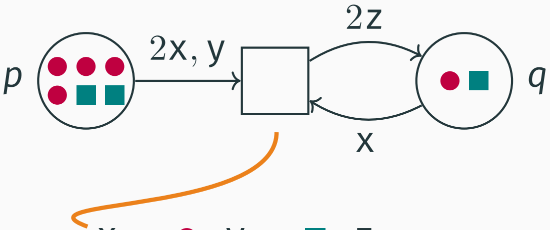
Unordered data Petri nets (UDPNs)



Marking

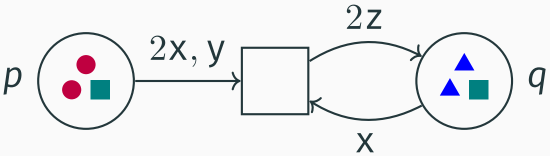
($\mathbf{M} \colon \mathbb{D} \times P \to \mathbb{N}$ with finite support)

Unordered data Petri nets (UDPNs)



x := ●, y := ■, z := ▲,

Unordered data Petri nets (UDPNs)



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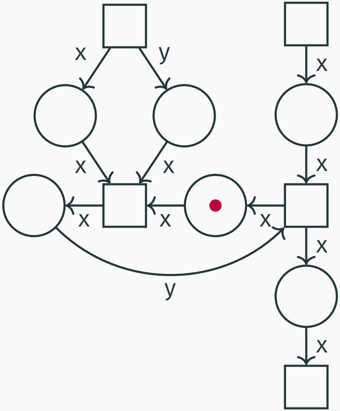
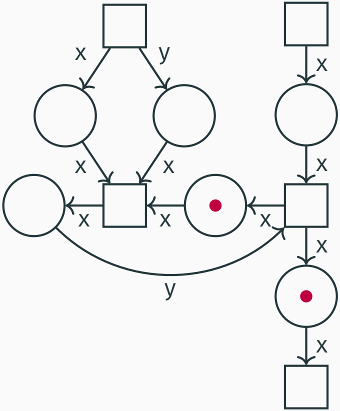


E.g. starting from this marking

Unordered data Petri nets (UDPNs)



Can we reach this one?

Reachability problem

Given: UDPN $\mathcal{N}$ and markings $\mathbf{M}_{\mathsf{init}}, \mathbf{M}_{\mathsf{tgt}}$

Decide: whether $\mathbf{M}_{\mathsf{init}} \xrightarrow{*} \mathbf{M}_{\mathsf{tgt}}$

Reachability

Reachability problem

Given: UDPN $\mathcal{N}$ and markings $\mathbf{M}_{\text{init}}, \mathbf{M}_{\text{tgt}}$

Decide: whether $\mathbf{M}_{\text{init}} \xrightarrow{*} \mathbf{M}_{\text{tgt}}$

Decidable if all tokens black

Ackermann-complete

(Leroux, Schmitz LICS'19;
Czerwinski, Orlikowski FOCS'21; Leroux FOCS'21)

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In general, decidability is open!

Reachability problem

Given: UDPN $\mathcal{N}$ and markings $\mathbf{M}_{\text{init}}, \mathbf{M}_{\text{tgt}}$

Decide: whether $\mathbf{M}_{\text{init}} \xrightarrow{*} \mathbf{M}_{\text{tgt}}$

Undecidable
with fresh color creation
(Rosa-Velardo, de Frutos-Escrig TCS'11)

Reachability

Coverability problem

Given: UDPN $\mathcal{N}$ and markings $\mathbf{M}_{\text{init}}, \mathbf{M}_{\text{tgt}}$

Decide: whether $\mathbf{M}_{\text{init}} \xrightarrow{*} \mathbf{M}$ for some $\mathbf{M}_{\text{tgt}} \sqsubseteq \mathbf{M}$

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Decidable

in hyper-Ackermannian time

(Lazic, Newcomb, Ouaknine, Roscoe, Worrell ICATPN'07;
Hofman, Lasota, Lazic, Leroux, Schmitz, Totzke FoSSaCS'16)

Our quest

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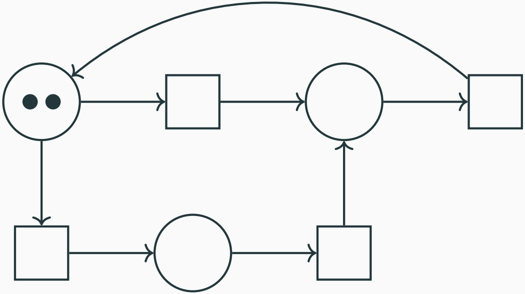
- UDPNs are one of the few extensions of Petri nets where reachability might be decidable
- Establishing decidability is ambitious…
- But, is there a subclass of UDPNs for which reachability is decidable?

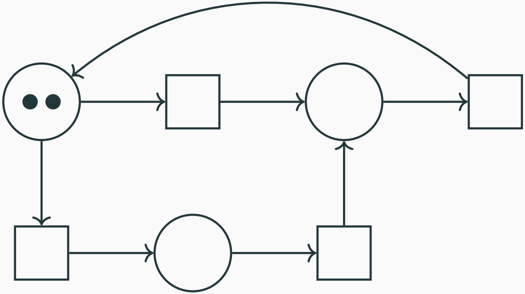
Let's try outrageous subclasses!

1st subclass: (dataless) S-nets



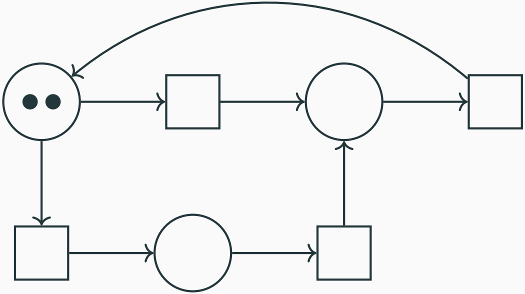
Each transition has one input and one output

1st subclass: (dataless) S-nets



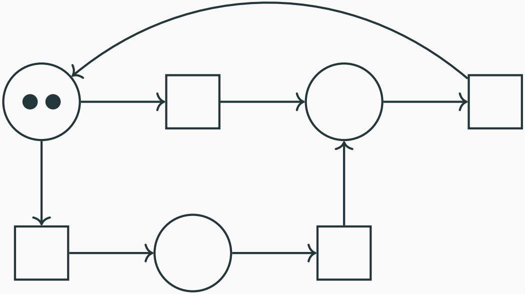
Each transition has one input and one output
(so the number of tokens never changes)

1st subclass: (dataless) S-nets



$$\boldsymbol{m} \xrightarrow{*} \boldsymbol{m}' \text{ iff } \boldsymbol{m} \xrightarrow{*}_{\mathbb{R}_{\geq 0}} \boldsymbol{m}'$$

1st subclass: (dataless) S-nets



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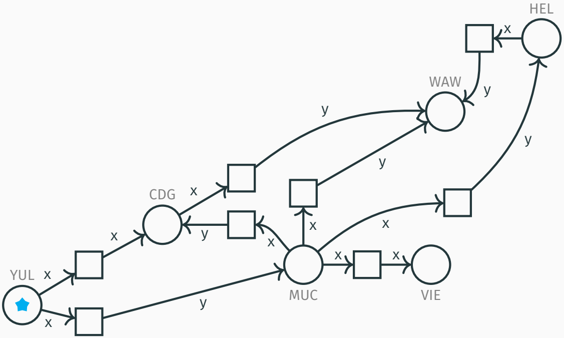
Polynomial time

1st subclass: unordered data S-nets



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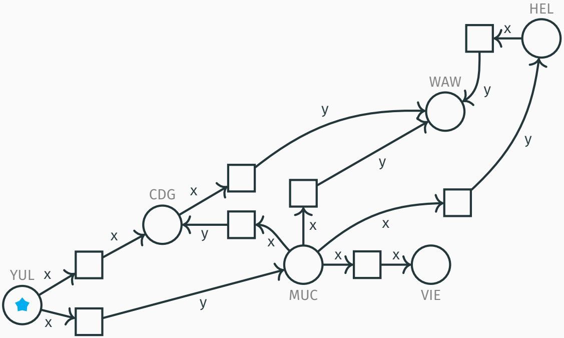
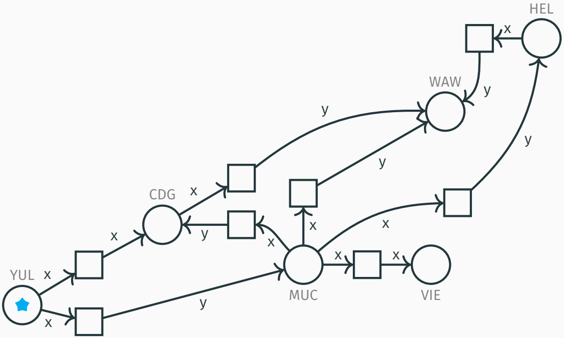


At most two auxiliary colors are needed to witness reachability

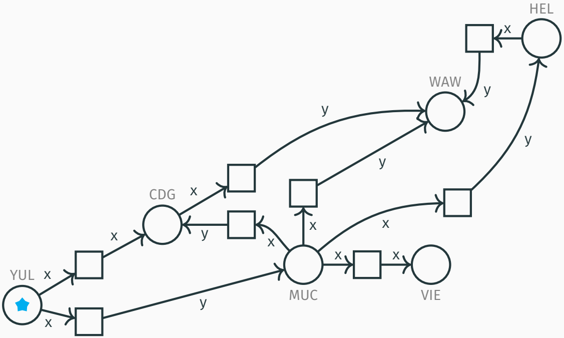
1st subclass: unordered data S-nets



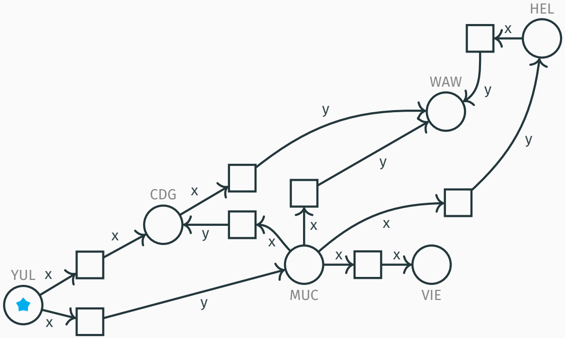
So, convert the net to

$|\mathrm{col}(\mathbf{M}_{\text{init}}) \cup \mathrm{col}(\mathbf{M}_{\text{tgt}})| + 2$ dataless copies

1st subclass: unordered data S-nets



The resulting net is still an *S*-net



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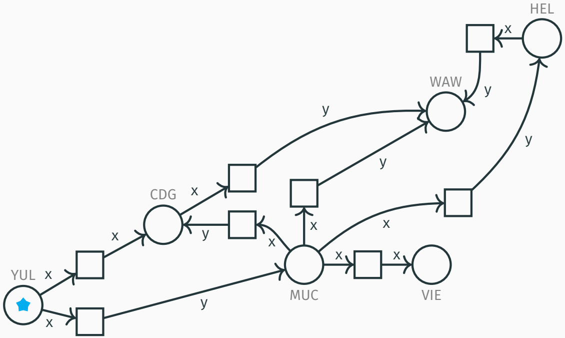
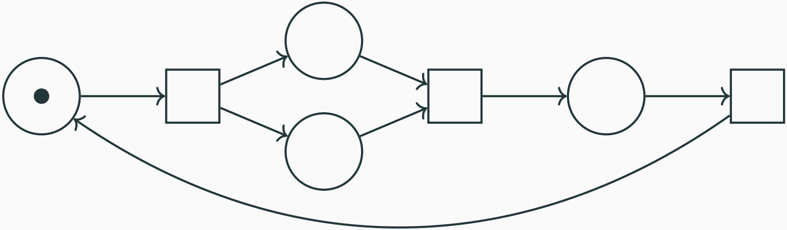
1st subclass: unordered data S-nets



So, reachability is in P

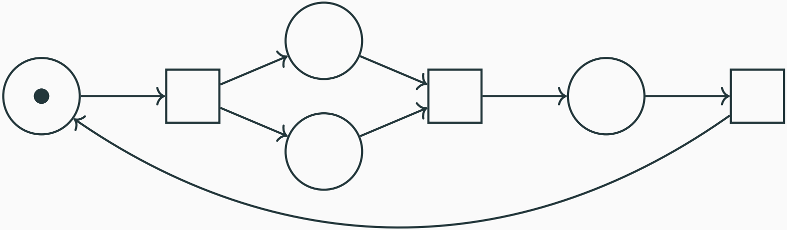
Great, let's continue!

2nd subclass: (dataless) T-nets



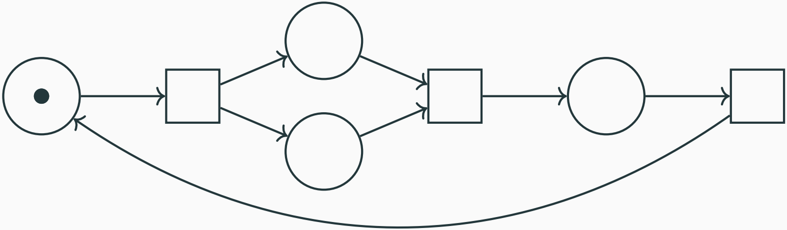
Each place has one input and one output

2nd subclass: (dataless) T-nets



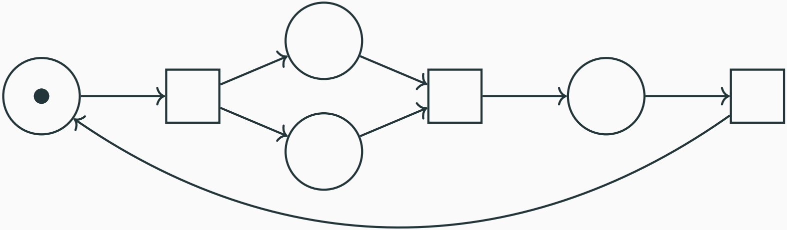
Each place has one input and one output

(so the number of tokens never changes in a cycle)



$$\boldsymbol{m} \xrightarrow{*} \boldsymbol{m}' \text{ iff } \exists \boldsymbol{w} : \boldsymbol{m} \xrightarrow{\boldsymbol{w}}_{\mathbb{R}_{\geq 0}} \boldsymbol{m}' \text{ and } \boldsymbol{w}(t) = 0$$

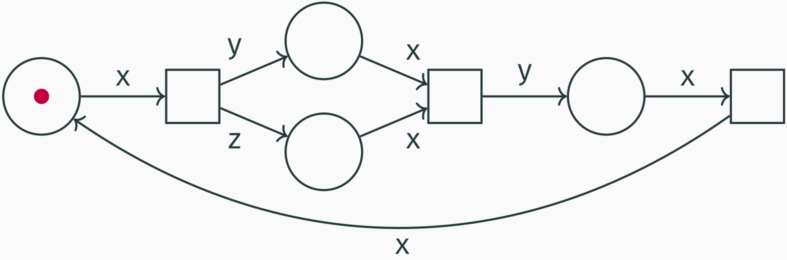
for all t in an unmarked cycle



$\boldsymbol{m} \xrightarrow{*} \boldsymbol{m}'$ iff $\exists \boldsymbol{w} : \boldsymbol{m} \xrightarrow{\boldsymbol{w}}_{\mathbb{R}_{\geq 0}} \boldsymbol{m}'$ and $\boldsymbol{w}(t) = 0$ for all t in an unmarked cycle

Polynomial time

2nd subclass: unordered data T-nets



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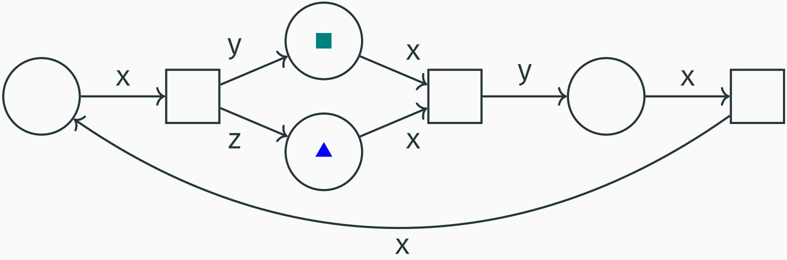

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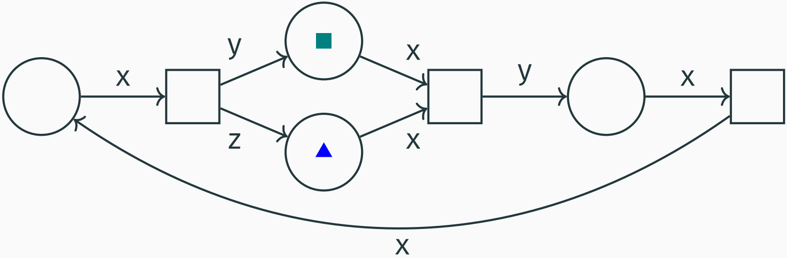
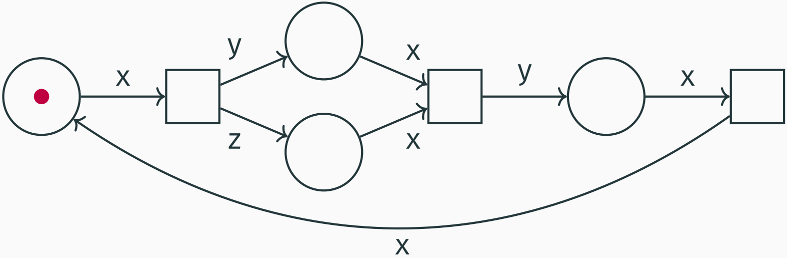


Stuck!

But what if tokens could split?

2nd subclass: unordered data T-nets



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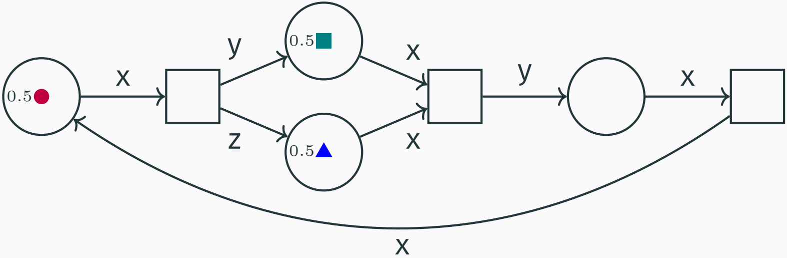

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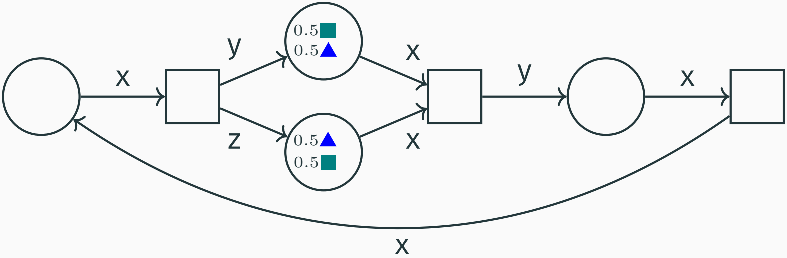

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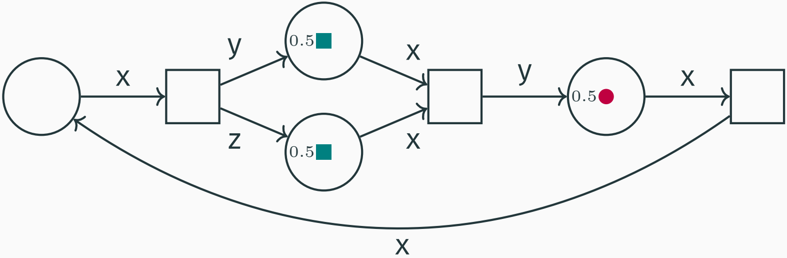

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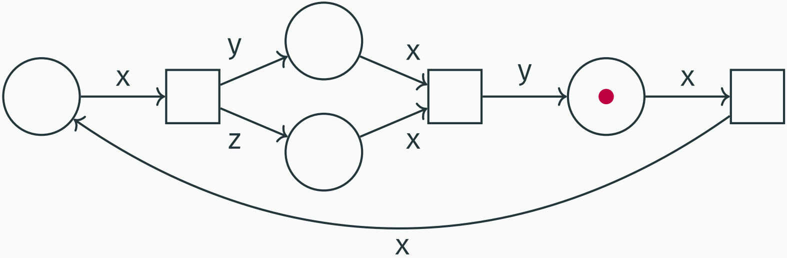

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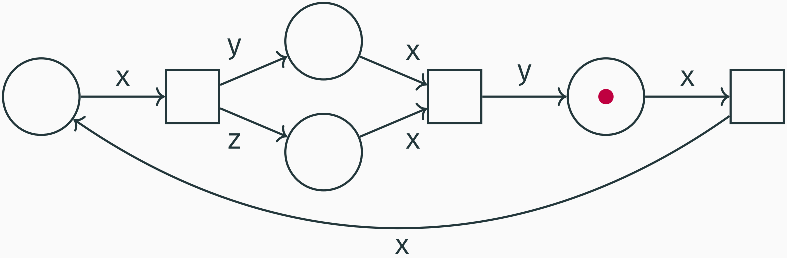


Reasoning over $\mathbb{R}_{\geq 0}$ not working!

Unordered data T-nets are harder

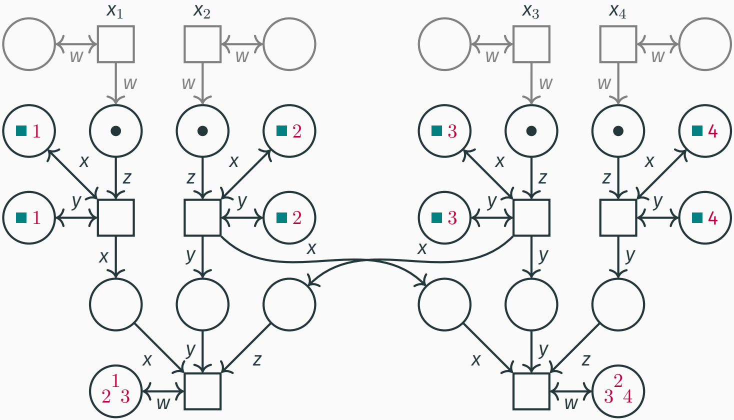
Proposition

Reachability in unordered data T-nets is NP-hard

Proof

Reduction from 1-in-3 3-SAT

Unordered data T-nets are harder



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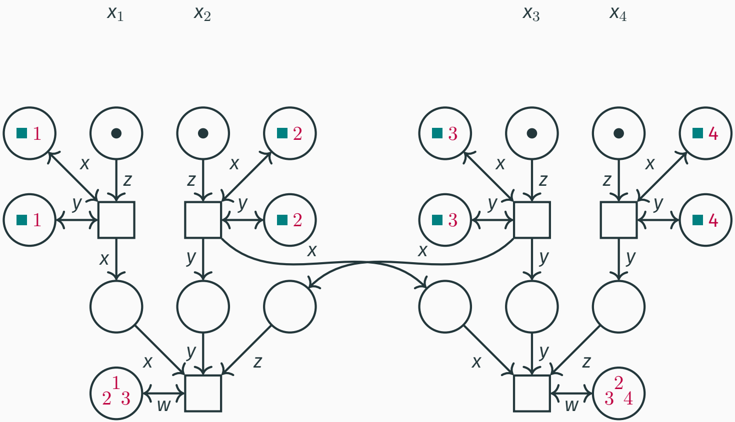

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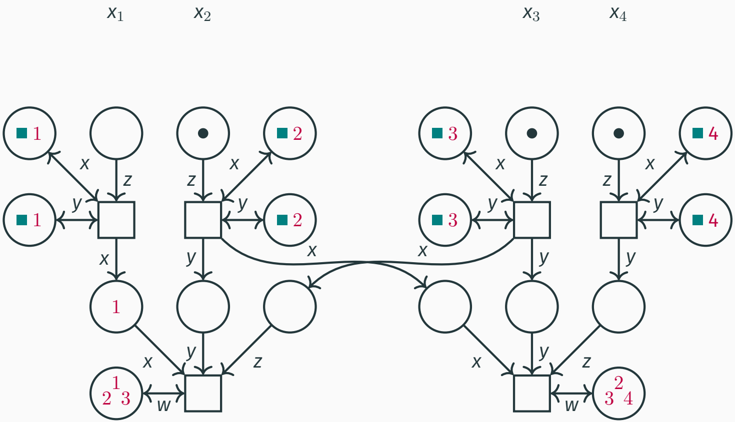

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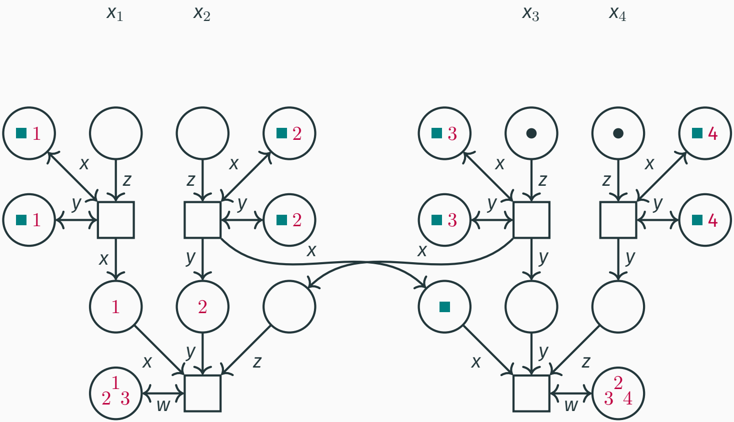

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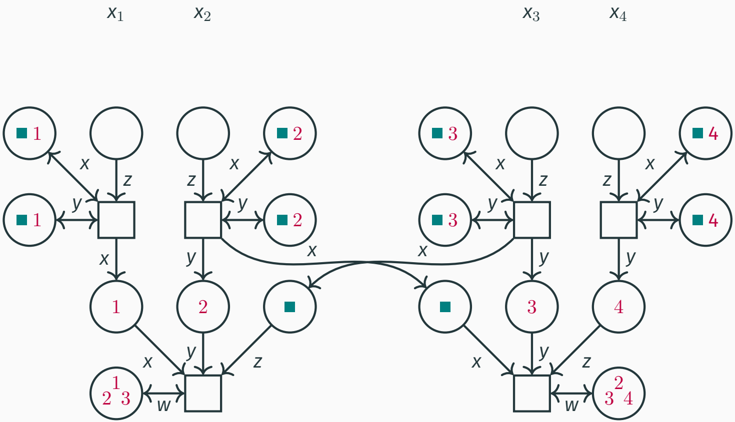

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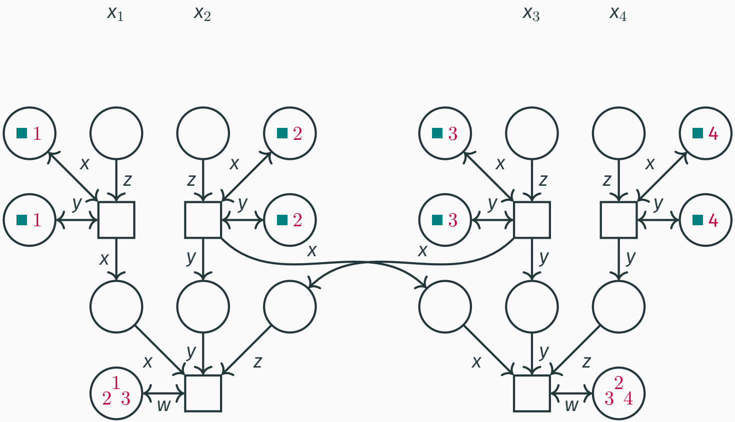

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Unordered data T-nets: decidability?

Is reachability decidable?

Proposition

Reachability in unordered data T-nets is

- $\in$ PSPACE for strongly connected nets
- $\in$ P for acyclic nets

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- So, reachability $\in$ NPSPACE $=$ PSPACE

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  - $\mathbf{X} := \mathbf{M}' - \mathbf{M}$
  - $\sum_{d \in \mathbb{D}} \mathbf{X}(d)$ is in the subgroup of $(\mathbb{Z}^P, +)$ generated by $\left\{\sum_{x \in Var} \mathbf{\Delta}_t(x) : t \in T\right\}$, and
  - For each $d \in \mathbb{D}$, $\mathbf{X}(d)$ is in the subgroup of $(\mathbb{Z}^P, +)$ generated by $\{\mathbf{\Delta}_t(x) : t \in T, x \in Var\}$

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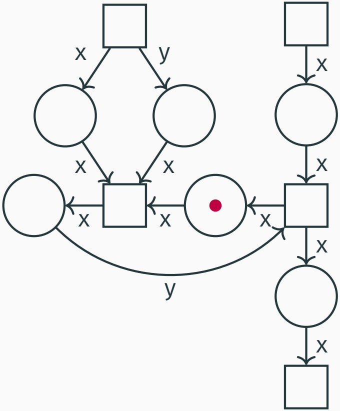
$\in P$

by Gupta, Shah, Akshay, Hofman FoSSaCS'19

Unordered data T-nets: general case



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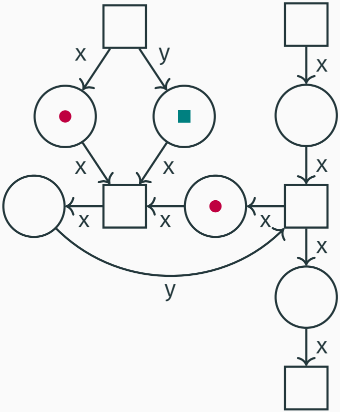

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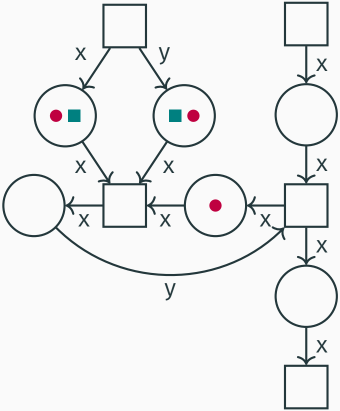

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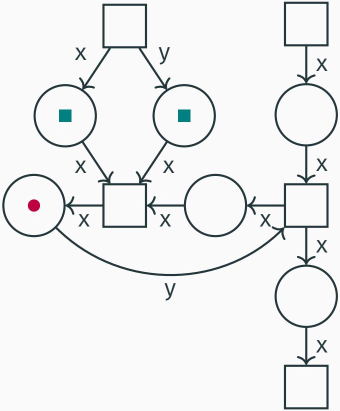

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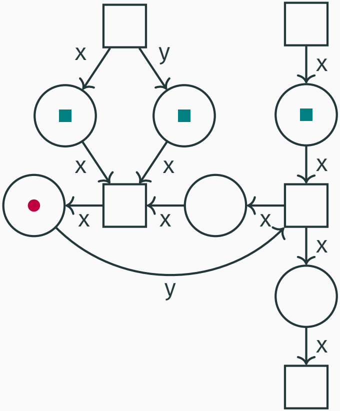

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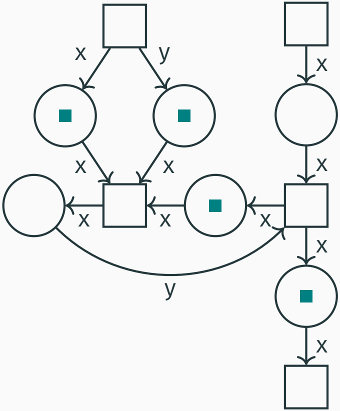

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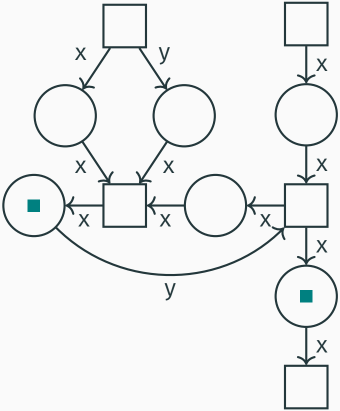

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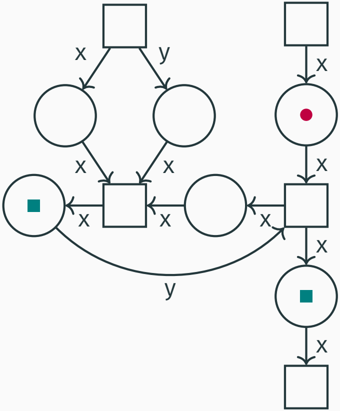

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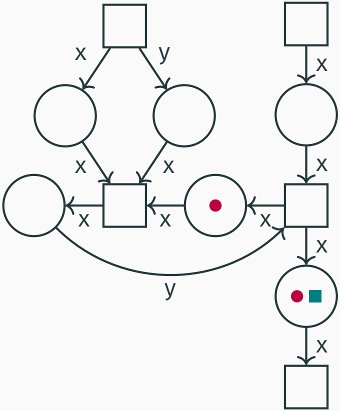

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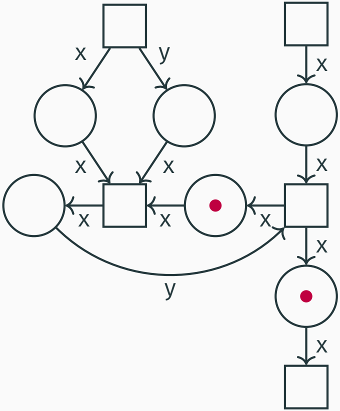

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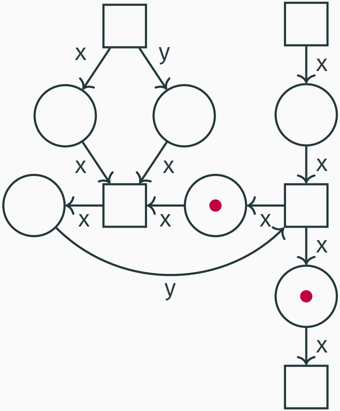

How to determine this algorithmically?

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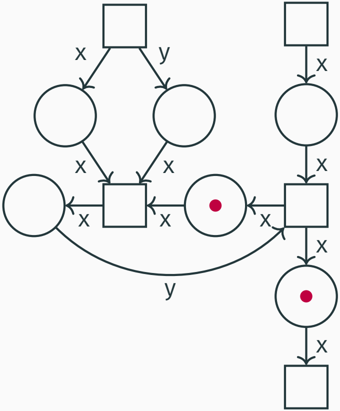

No idea... we've been stuck here!

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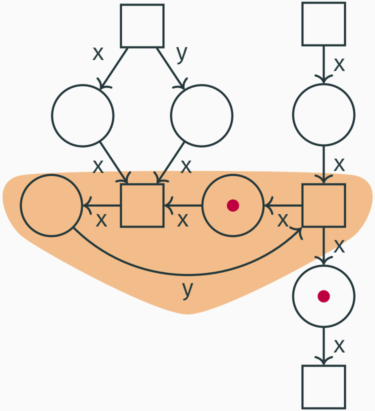

Symbolic representation of SCCs?

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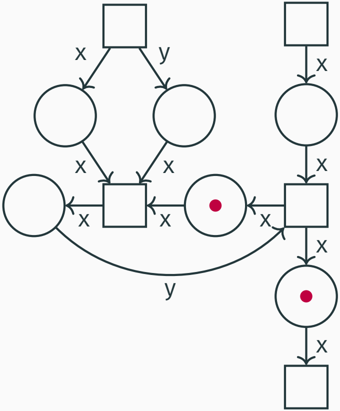

Bound number of colors?

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- Some conjecture decidability to hold
- But we failed to establish it for the outrageously limited case of T-nets
- Maybe for another natural subclass... But still, why not T-nets?!

Thank you!

Dziękuję!