Approaching the Coverability Problem Continuously

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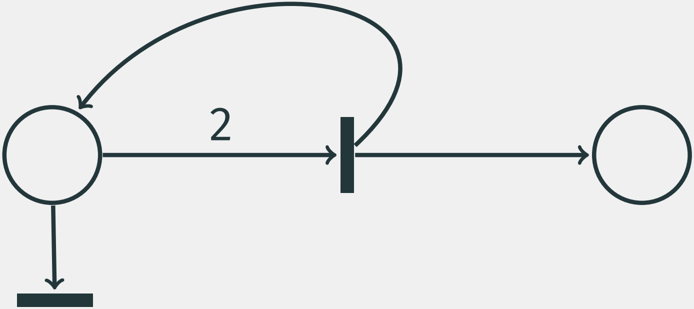
(Discrete) Petri nets



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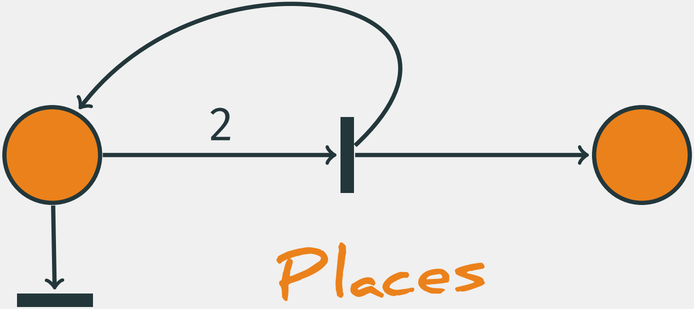

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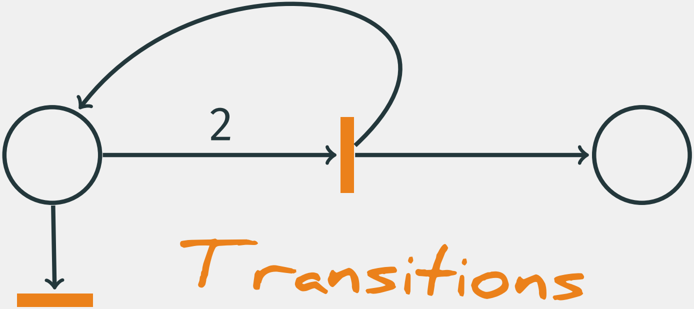

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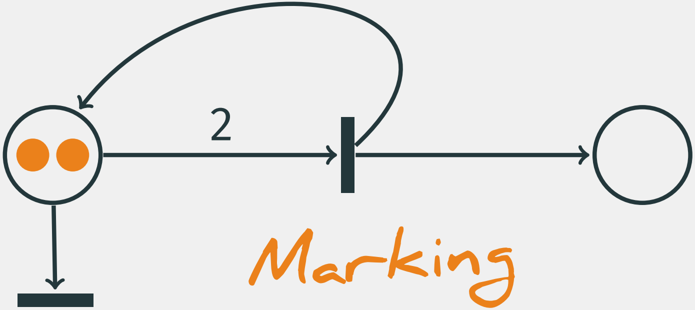

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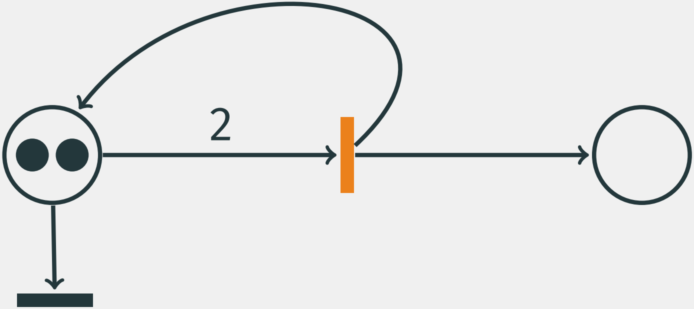

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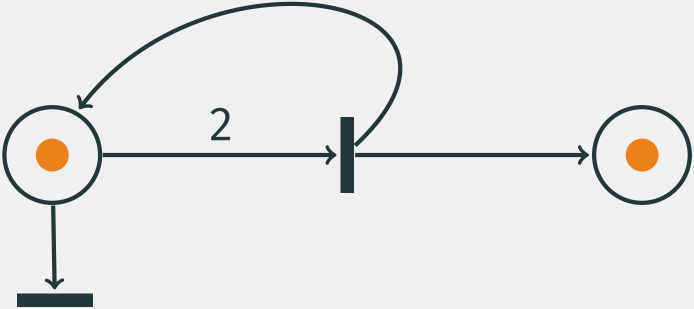

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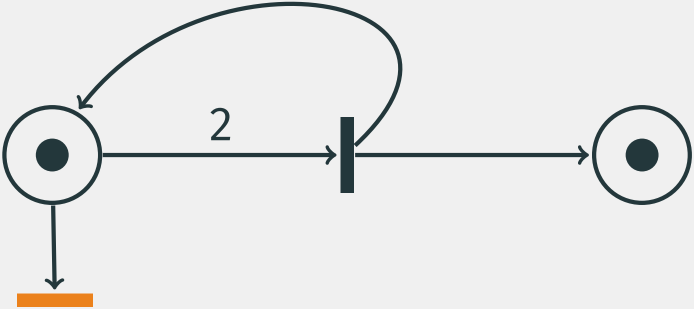

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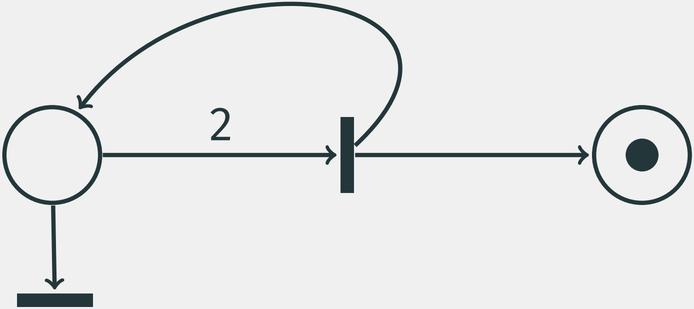

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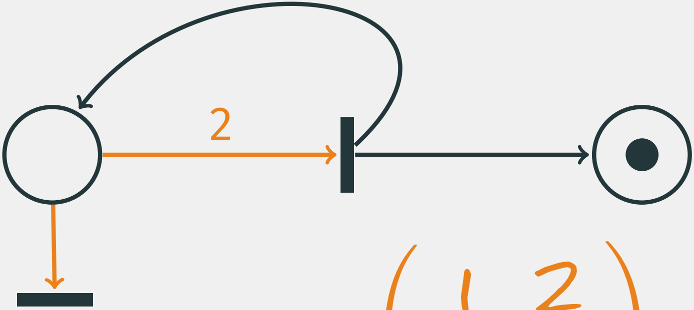

$$Pre = \begin{pmatrix} 1 & 2 \\ 0 & 0 \end{pmatrix}$$

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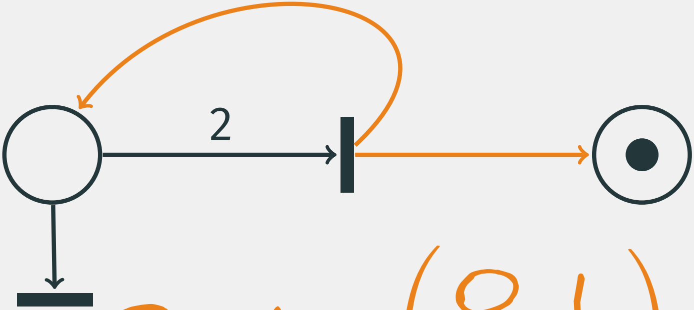

$$Post = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}$$

Lamport mutual exclusion "1-bit algorithm"

Verifying safety with Petri nets

Process 1

Process 2

Lamport mutual exclusion "1-bit algorithm"

Lamport mutual exclusion "1-bit algorithm"

Verifying safety with Petri nets

```
while True:
  x = True
  while y: pass
  # critical section
  x = False
```

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while True:
★ y = True
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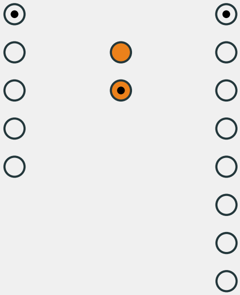

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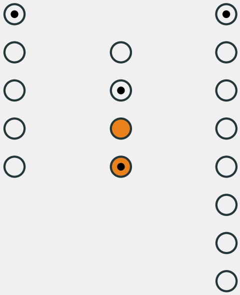

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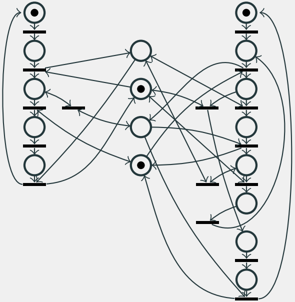

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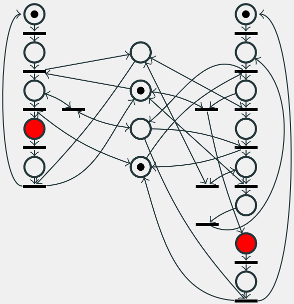

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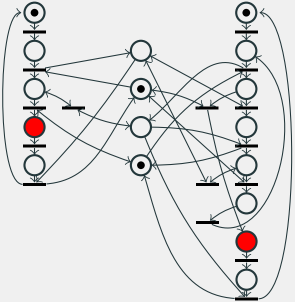

Processes at both critical sections $\iff$ each  ≥ 1

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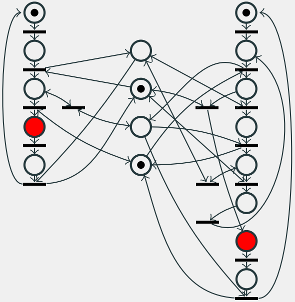

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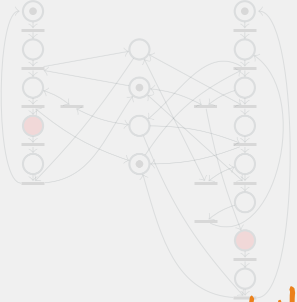
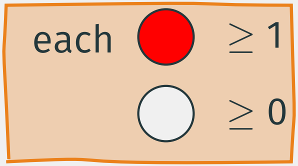
 ≥ 0



Coverability problem

Processes at both critical sections $\iff$



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Problem

Input:	Petri net $\mathcal{N}$, initial marking $\boldsymbol{m}_0$, target marking $\boldsymbol{m}$
Question:	Is some $\boldsymbol{m}' \geq \boldsymbol{m}$ reachable from $\boldsymbol{m}_0$ in $\mathcal{N}$?

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How to solve it?

- Forward: build reachability tree from initial marking
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- EXPSPACE-complete

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How to solve it? **Karp & Miller '69**

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How to solve it? **Arnold & Latteux '78, Abdulla *et al.* '96**

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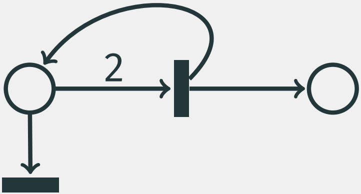

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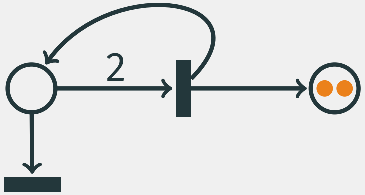
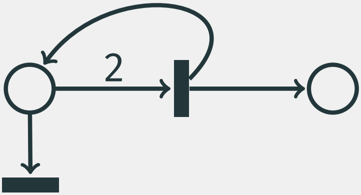
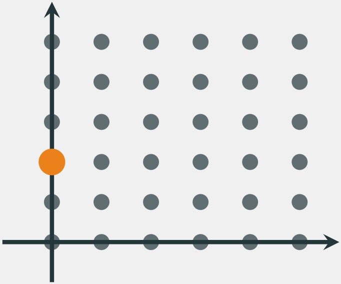


What initial markings may cover $(0, 2)$?

Backward algorithm





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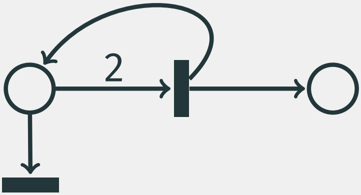

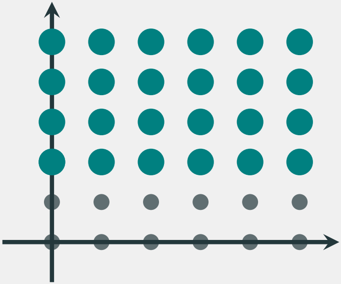

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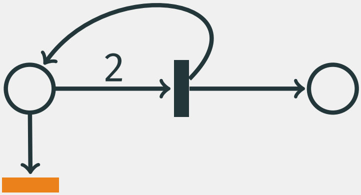

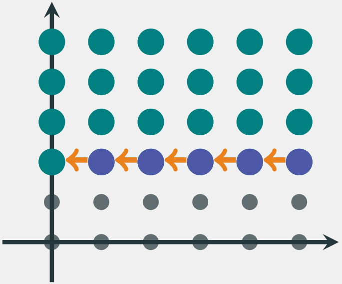

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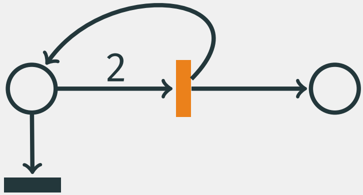

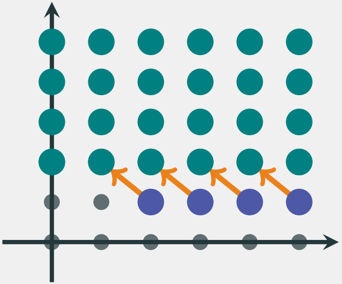

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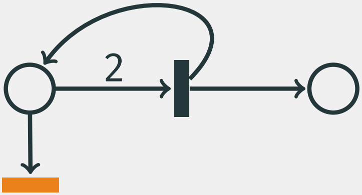

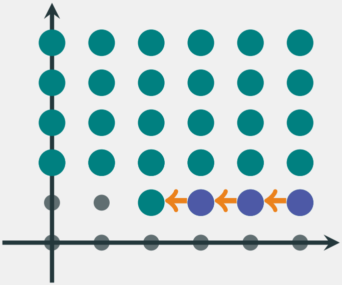

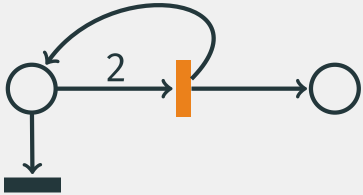
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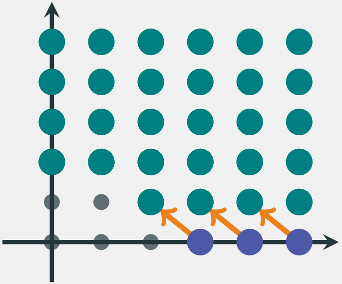

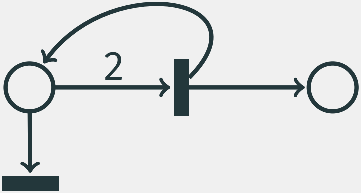
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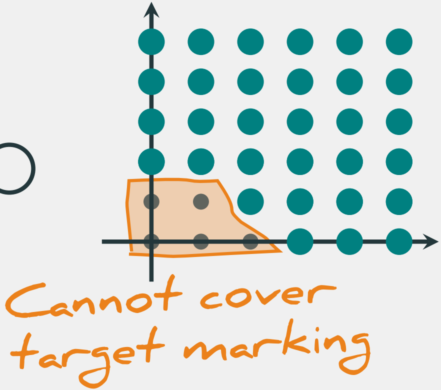
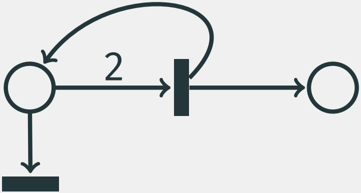
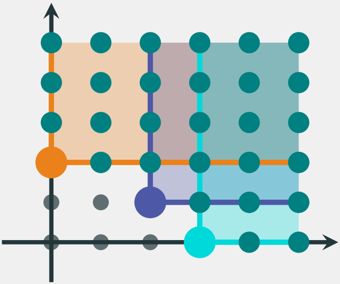

Cannot cover
target marking

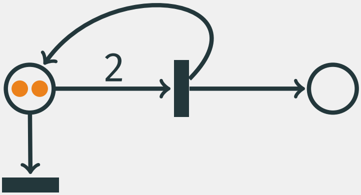


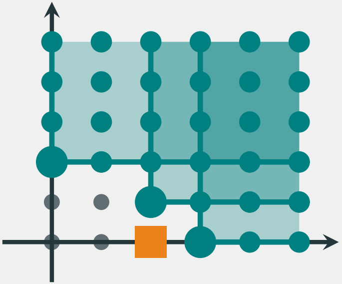


Basis size may become doubly exponential

(Bozzelli & Ganty '11)

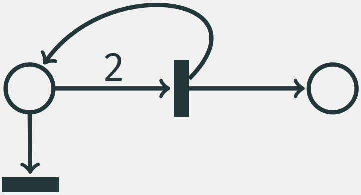
Backward algorithm

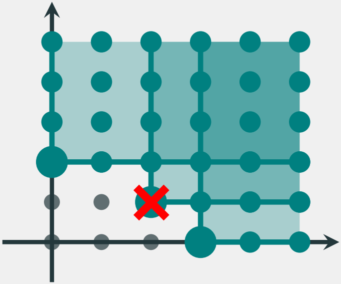




We only care about some initial marking...

Backward algorithm





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Speedup by pruning basis!

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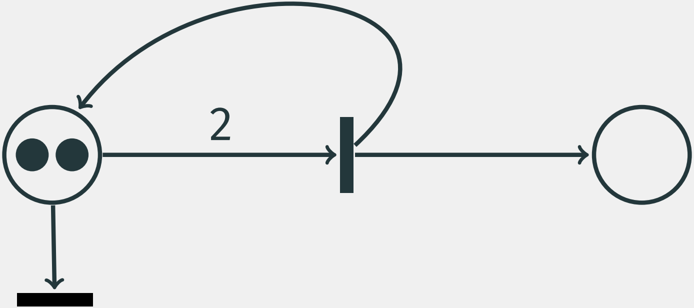

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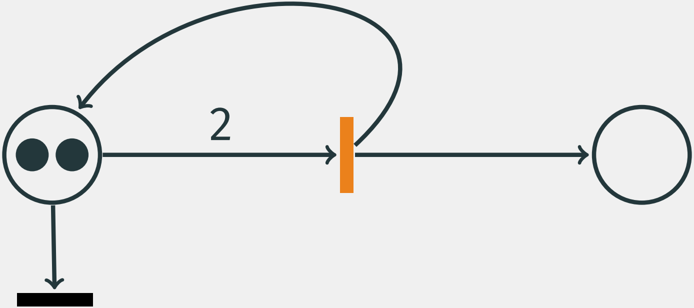

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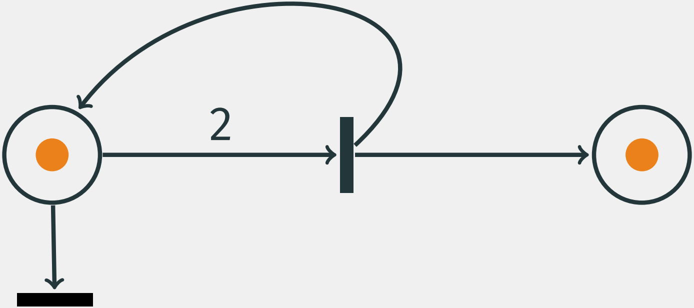

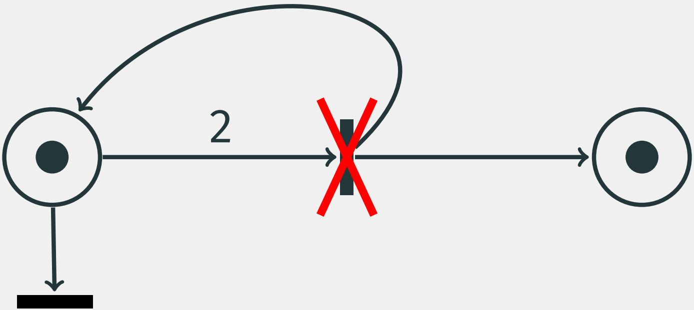
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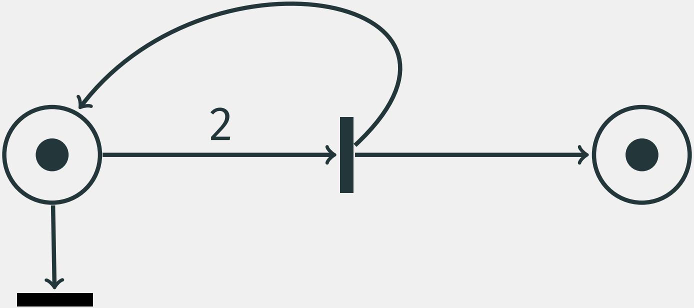

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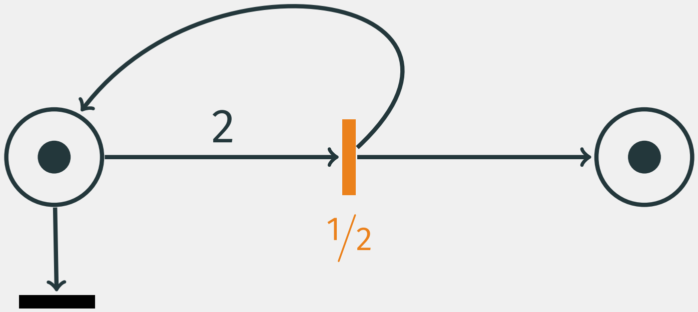

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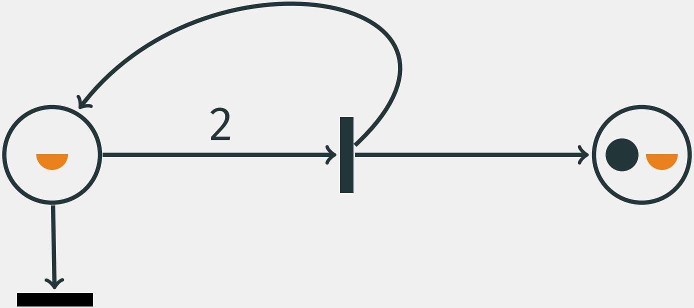

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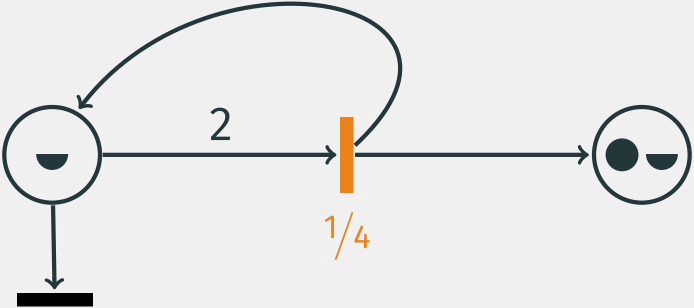

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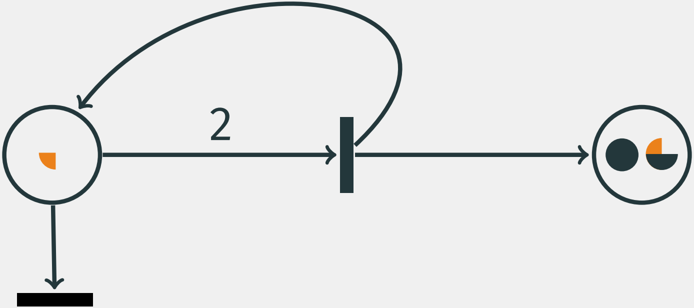

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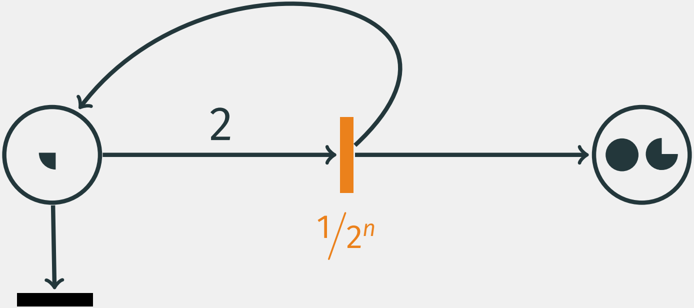

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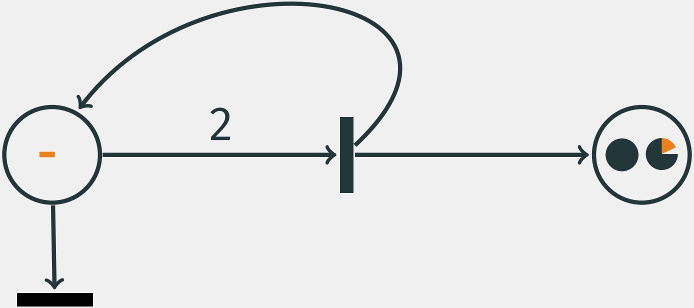

$\boldsymbol{m}$ is coverable from $\boldsymbol{m}_0$

$$\Downarrow$$

$\boldsymbol{m}$ is $\mathbb{Q}$-coverable from $\boldsymbol{m}_0$

Continuity to over-approximate coverability

$\boldsymbol{m}$ is coverable from $\boldsymbol{m}_0$

EXPSPACE

$\Downarrow$

$\boldsymbol{m}$ is $\mathbb{Q}$-coverable from $\boldsymbol{m}_0$

$\Downarrow \quad \not\Uparrow$

PTIME

$\boldsymbol{m}_0$ and $\boldsymbol{m}$ satisfy conditions of

Esparza, Ledesma-Garza, Majumdar, Meyer & Niksic '14

PTIME / NP / coNP

$\boldsymbol{m}$ is not coverable from $\boldsymbol{m}_0$

Safety

$\Uparrow$

$\boldsymbol{m}$ is not $\mathbb{Q}$-coverable from $\boldsymbol{m}_0$

Coverability in continuous Petri nets

Fix some continuous Petri net $(P, T, \mathbf{Pre}, \mathbf{Post})$

m is $\mathbb{Q}$-coverable from m_0 iff... **Fraca & Haddad '13**

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there exist $\boldsymbol{m}' \geq \boldsymbol{m}$ and $\boldsymbol{v} \in \mathbb{Q}^{T}_{\geq 0}$ such that

- $\boldsymbol{m}' = \boldsymbol{m_0} + (\mathbf{Post} - \mathbf{Pre}) \cdot \boldsymbol{v}$

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- some execution from $\boldsymbol{m}_0$ fires exactly $\{t \in T : \boldsymbol{v}_t > 0\}$

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- $\boldsymbol{m}' = \boldsymbol{m}_0 + (\mathbf{Post} - \mathbf{Pre}) \cdot \boldsymbol{v}$
- some execution from $\boldsymbol{m}_0$ fires exactly $\{t \in T : \boldsymbol{v}_t > 0\}$
- some execution to $\boldsymbol{m}'$ fires exactly $\{t \in T : \boldsymbol{v}_t > 0\}$

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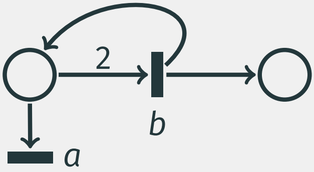

$\boldsymbol{m}_0 = (2, 0)$

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$\boldsymbol{m}$ is $\mathbb{Q}$-coverable from $\boldsymbol{m}_0$ iff... **Fraca & Haddad '13**

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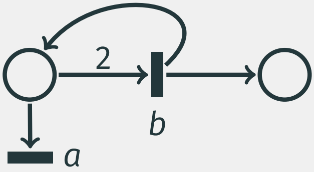
Coverability in continuous Petri nets



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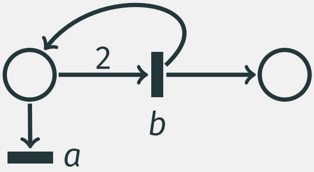

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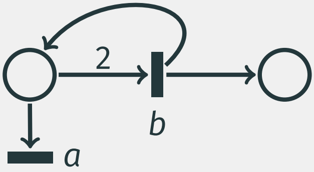

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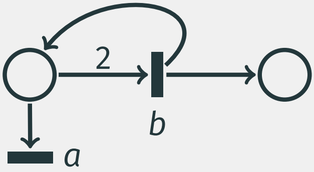

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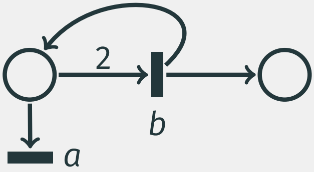
Coverability in continuous Petri nets



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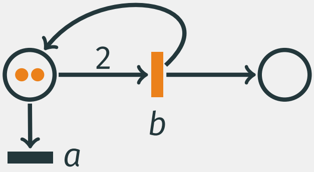
Coverability in continuous Petri nets



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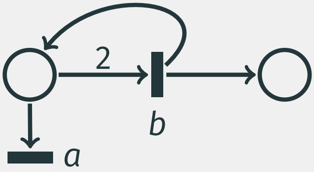

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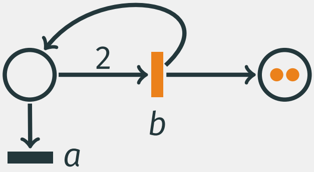

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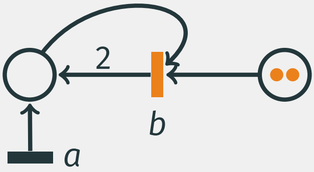
Coverability in continuous Petri nets



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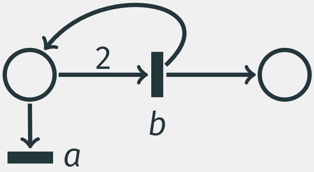
Coverability in continuous Petri nets



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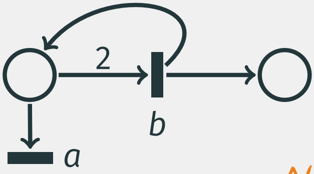
Coverability in continuous Petri nets



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Not $\mathbb{Q}$-coverable from

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Coverability in continuous Petri nets

Polynomial time!

$\boldsymbol{m}$ is $\mathbb{Q}$-coverable from $\boldsymbol{m}_0$ iff... **Fraca & Haddad '13**

there exist $\boldsymbol{m}' \geq \boldsymbol{m}$ and $\boldsymbol{v} \in \mathbb{Q}_{\geq 0}^{T}$ such that

- $\boldsymbol{m}' = \boldsymbol{m}_0 + (\mathbf{Post} - \mathbf{Pre}) \cdot \boldsymbol{v}$
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Coverability in continuous Petri nets

Logical characterization **Contribution**

$\mathbb{Q}$-coverability can be encoded in a linear size formula of

existential $\mathsf{FO}(\mathbb{Q}_{\geq 0}, +, <)$

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Coverability in continuous Petri nets

Logical characterization **Contribution**

$\mathbb{Q}$-coverability can be encoded in a linear size formula of

$$\text{existential FO}(\mathbb{N}, +, <)$$

Even better approximation

***m* is $\mathbb{Q}$-coverable from m_0 iff...** **Fraca & Haddad '13**

there exist $\boldsymbol{m}' \geq \boldsymbol{m}$ and $\boldsymbol{v} \in \mathbb{Q}^T_{\geq 0}$ such that

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Coverability in continuous Petri nets

Logical characterization

Contribution

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Fraca & Haddad '13

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- $\boldsymbol{m}' = \boldsymbol{m}_0 + (\mathbf{Post} - \mathbf{Pre}) \cdot \boldsymbol{v}$ ← Straightforward
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Coverability in continuous Petri nets

Logical characterization — Contribution

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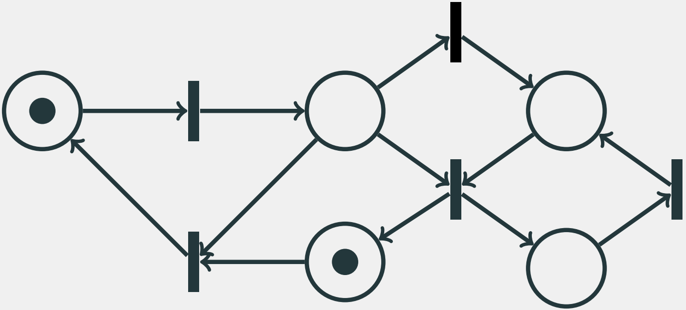
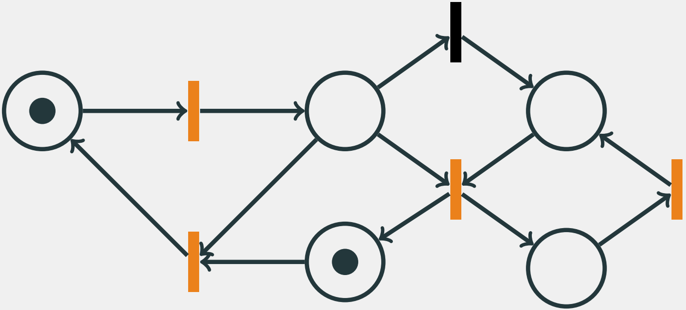
More subtle

Encoding the firing set conditions



Testing whether some transitions can be fired from initial marking

Encoding the firing set conditions



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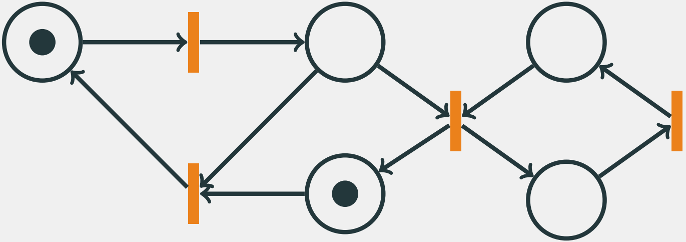

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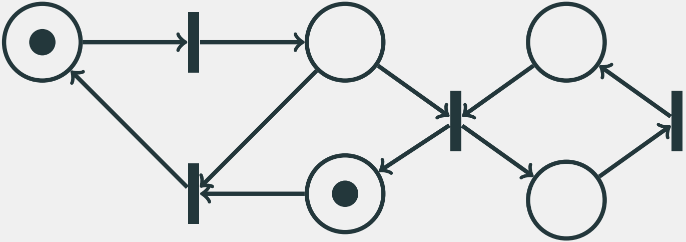
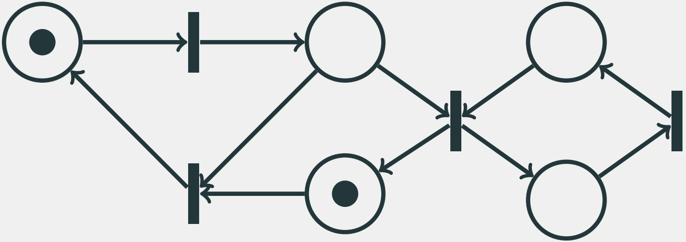


Simulate a "breadth-first" transitions firing

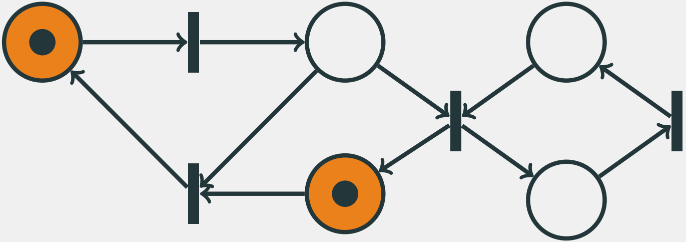
Encoding the firing set conditions



Simulate a "breadth-first" transitions firing

by numbering places/transitions

(Verma, Seidl & Schwentick '05)



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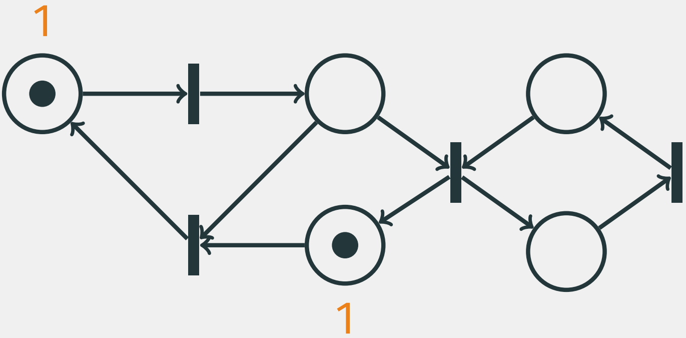

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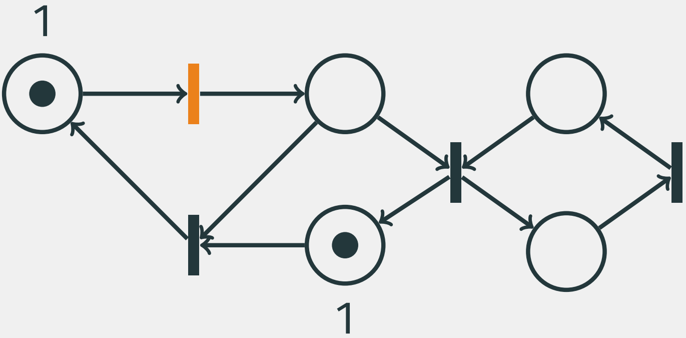

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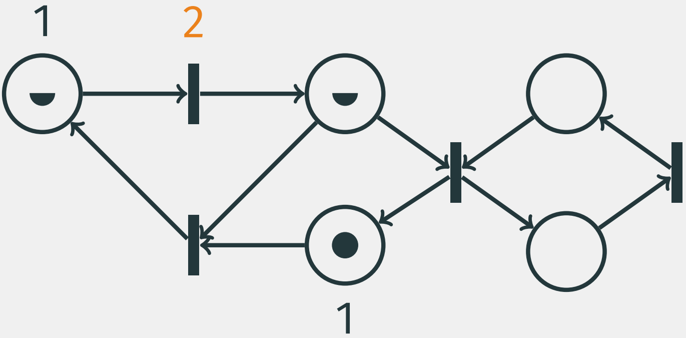

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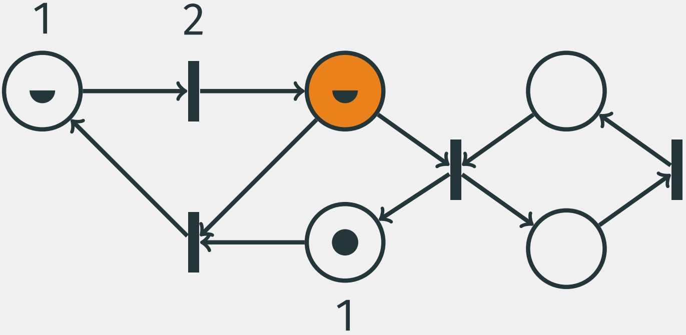

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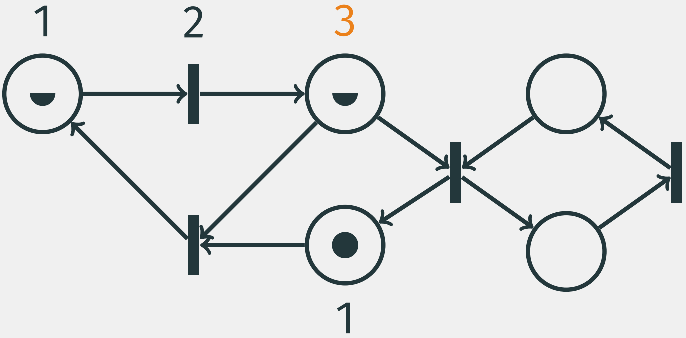

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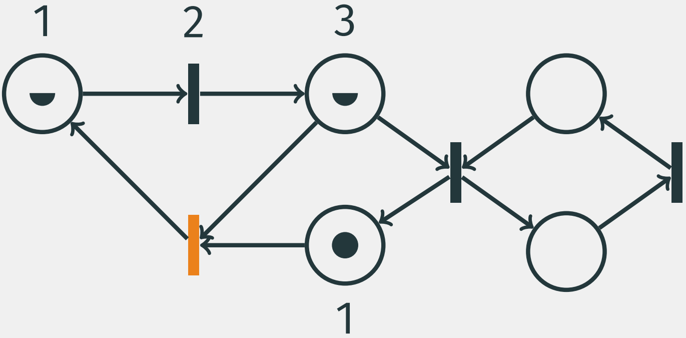

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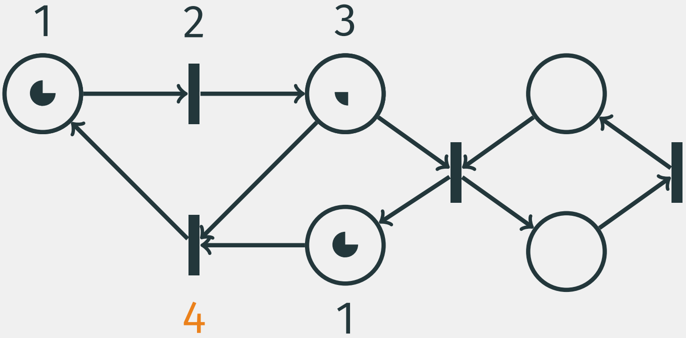

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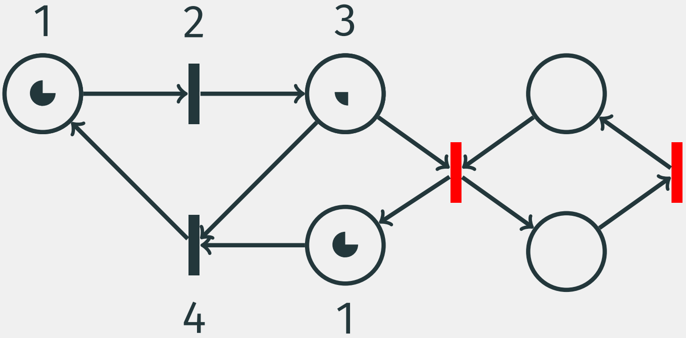

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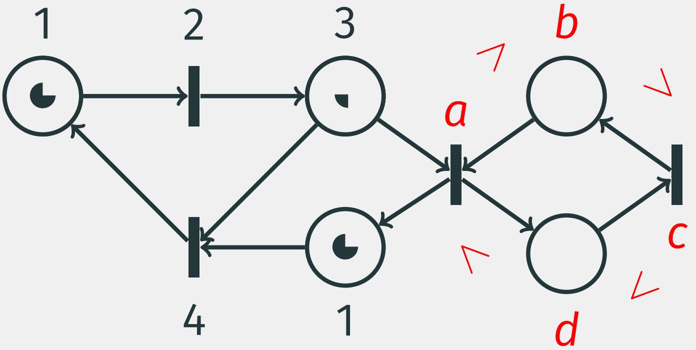

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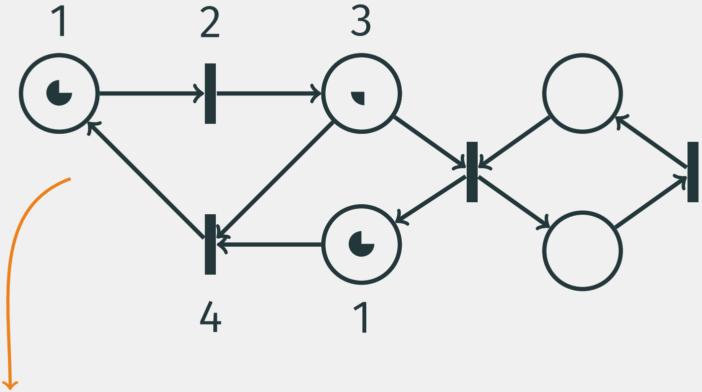

$$\varphi(\boldsymbol{x}) = \exists \boldsymbol{y} : \bigwedge_{p \in P} \boldsymbol{y}(p) > 0 \rightarrow \bigwedge_{t \in {}^\bullet p} \boldsymbol{y}(t) < \boldsymbol{y}(p) \cdots$$

Backward coverability modulo $\mathbb{Q}$-coverability

```
if target marking m is not ℚ-coverable:
  return False
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Polynomial time

Backward coverability modulo $\mathbb{Q}$-coverability

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if target marking m is not Q-coverable:
  return False
X = {target marking m}
while (initial marking m_0 not covered by X):
  B = markings obtained from X one step backward
  B = B \ {b ∈ B : ¬φ(b)}
  if B = ∅: return False
  φ(x) = φ(x) ∧ ⋀_{pruned b} x ≱ b
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$X = \{\text{target marking } \boldsymbol{m}\}$

while (initial marking $\boldsymbol{m}_0$ not covered by X):

  B = markings obtained from X one step backward

  $B = B \setminus \{\boldsymbol{b} \in B : \neg\varphi(\boldsymbol{b})\}$ ← $\mathbb{Q}$-coverability pruning (better than poly. time)

  if $B = \emptyset$: **return** False

  $\varphi(\boldsymbol{x}) = \varphi(\boldsymbol{x}) \ \wedge \ \bigwedge_{\text{pruned } \boldsymbol{b}} \boldsymbol{x} \not\geq \boldsymbol{b}$

  $X = X \cup B$

return True

Backward coverability modulo $\mathbb{Q}$-coverability

```
if target marking m is not Q-coverable:
  return False
X = {target marking m}
while (initial marking m_0 not covered by X):
  B = markings obtained from X one step backward
  B = B \ {b ∈ B : ¬φ(b)}
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SMT solver guidance

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- 760 lines of code
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- supports dense/sparse matrices through NUMPY/SCIPY
- experimental parallelism support

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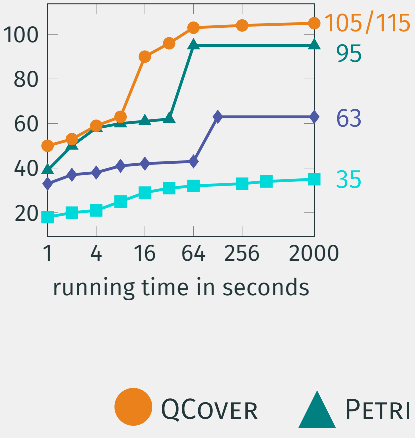
Benchmarks

Instances proven safe



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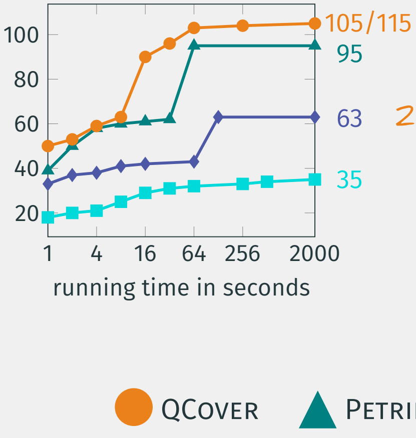


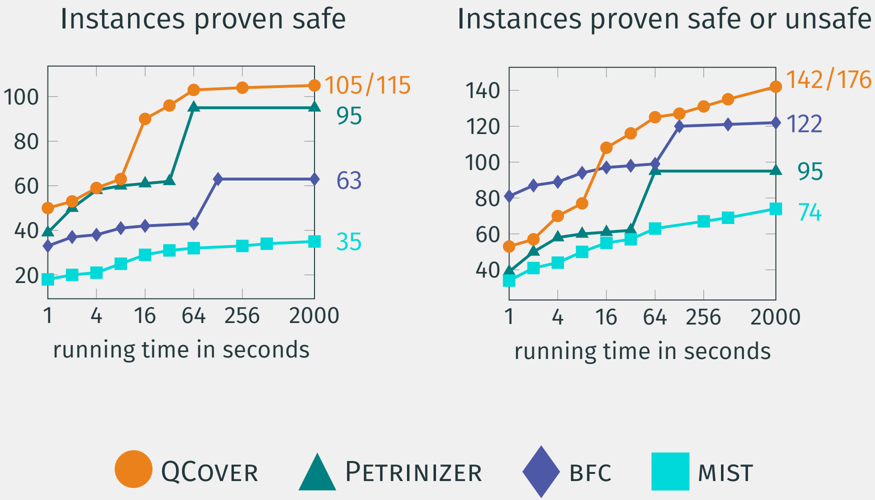
Largest nets proved safe:

21143 places
7150 trans.
42 secs.

6690 places
11934 trans.
21 secs.

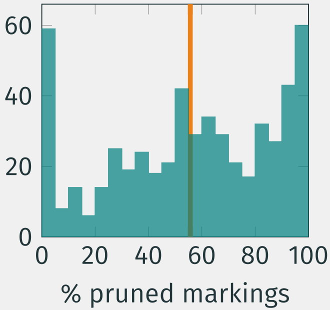
754 places
27370 trans.
3 secs.

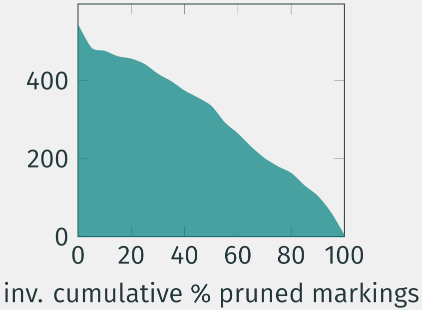
Benchmarks


Instances proven safe
100
80
60
40
20
1
4
16
64
256
2000
running time in seconds
105/115
95
63
35
Instances proven safe or unsafe
140
120
100
80
60
40
1
4
16
64
256
2000
running time in seconds
142/176
122
95
74
QCover
Petrinizer
BFC
MIST

Benchmarks

Markings pruning efficiency across all iterations





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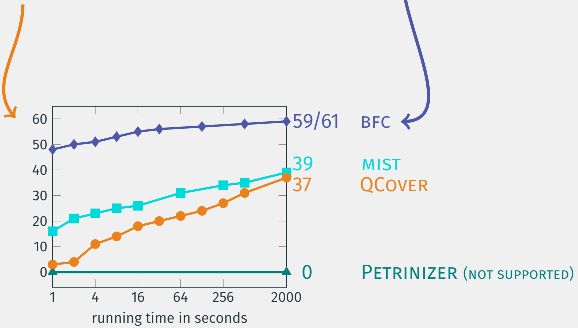

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- Use more efficient data structures, *e.g.* sharing trees

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- Use more efficient data structures, *e.g.* sharing trees

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- Extend to Petri nets with transfer/reset arcs

Thank you! Dank u!