The complexity of linear temporal verification for continuous counter systems

Michael Blondin

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Joint work with Alex Sansfaçon-Buchanan and Philip Offtermatt

Slides based on those of Alex

Université de Sherbrooke

Constant-rate multi-mode systems (MMS) Alur et al. HSCC'12

An *MMS* is a finite set $M \subseteq \mathbb{R}^d$

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For example, $M = \{\boldsymbol{m}_1, \boldsymbol{m}_2, \boldsymbol{m}_3\}$ where

$\boldsymbol{m}_1$	$\boldsymbol{m}_2$	$\boldsymbol{m}_3$
$(1, 2)$	$(1, 0)$	$(-1, -1)$

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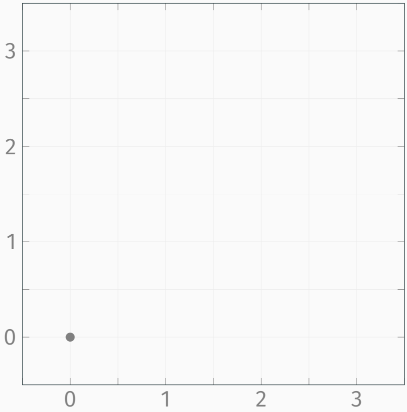
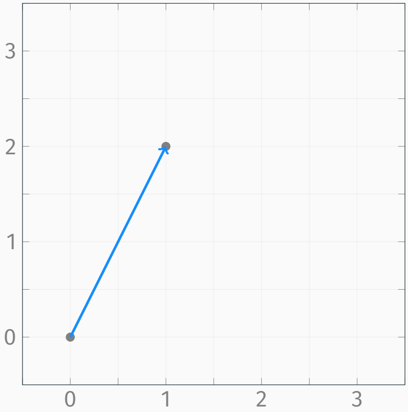
Introduced by Alur et al. to reason about problems related to green scheduling and energy peak-consumption reduction

Schedules and executions



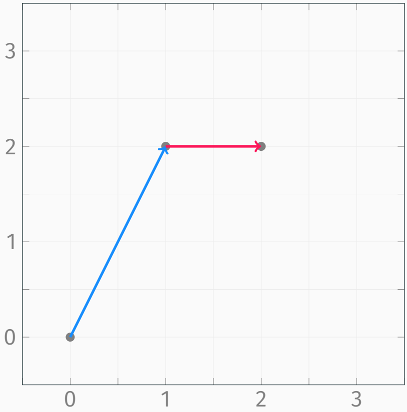
$\pi =$

Schedules and executions



$\pi = 1.0\boldsymbol{m}_1$

Schedules and executions



$\pi = 1.0\boldsymbol{m}_1 \; 1.0\boldsymbol{m}_2$

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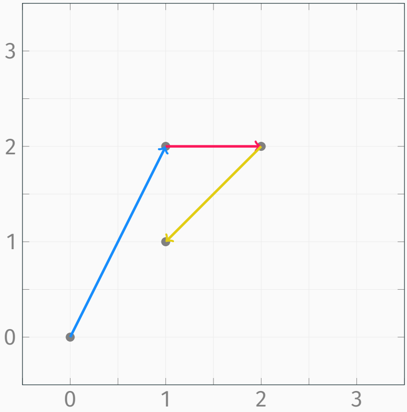

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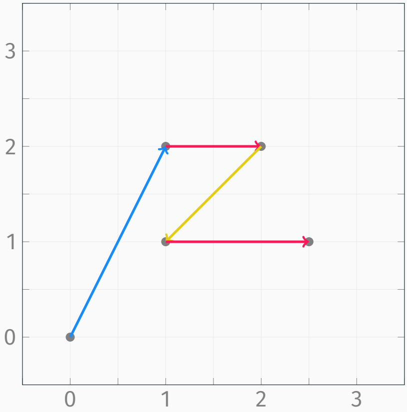
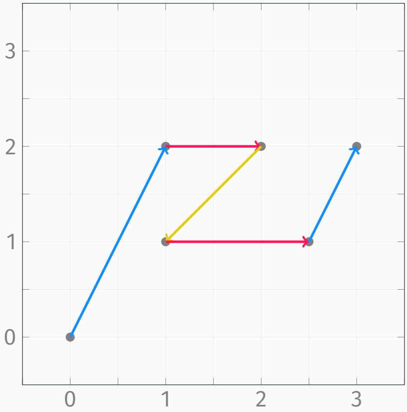


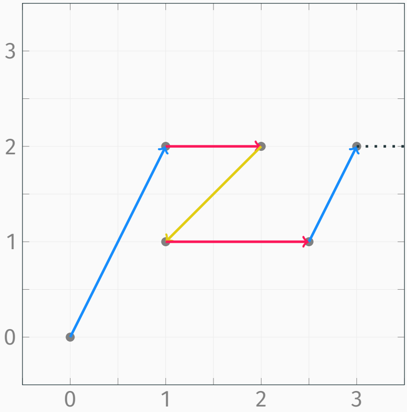
$\pi = 1.0\boldsymbol{m}_1\ 1.0\boldsymbol{m}_2\ 1.0\boldsymbol{m}_3\ 1.5\boldsymbol{m}_2$

Schedules and executions



$$\pi = 1.0\boldsymbol{m}_1 \; 1.0\boldsymbol{m}_2 \; 1.0\boldsymbol{m}_3 \; 1.5\boldsymbol{m}_2 \; 0.5\boldsymbol{m}_1$$

Schedules and executions



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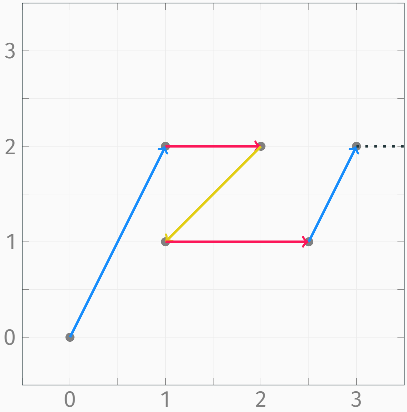
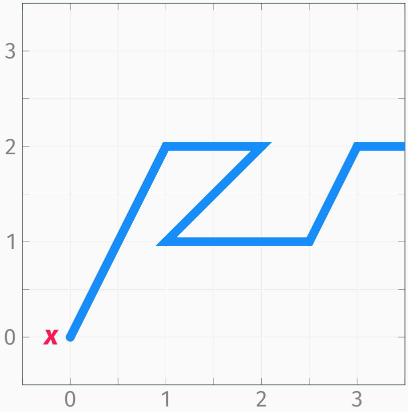


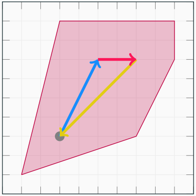
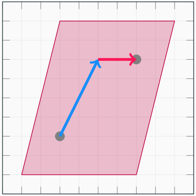
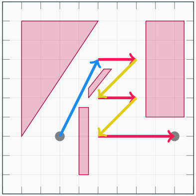
Schedule: $\pi = \alpha_1 \boldsymbol{m}_{i_1}\ \alpha_2 \boldsymbol{m}_{i_2}\ \cdots$ where $\alpha_i \in \mathbb{R}_{>0}$ and $\sum \alpha_i = \infty$

Schedules and executions



Execution: $\sigma : \mathbb{R}_{\geq 0} \to \mathbb{R}^d$ with $\sigma(0) = \boldsymbol{x}$

Known results

Safe scheduling	Safe reachability	Safe planning
Can always remain within a zone?	Can reach target within a zone?	Can reach target while avoiding obstacles?
		
PTIME-complete (Alur et al. HSCC'12)		Decidable (Krishna et al. ATVA'17)

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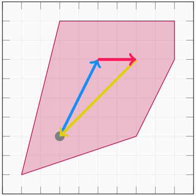	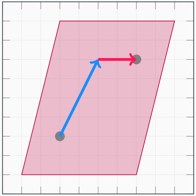	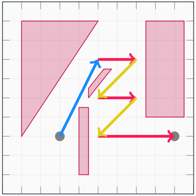
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* When each zone is a bounded closed convex polytope

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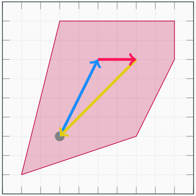	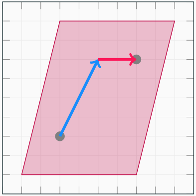	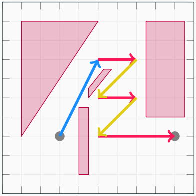
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* When each zone is a bounded closed convex polytope

$$\text{Zone} = \{\mathbf{x} \in \mathbb{R}^d : \mathbf{A}\mathbf{x} \leq \mathbf{b}\}$$

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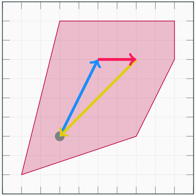	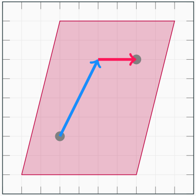	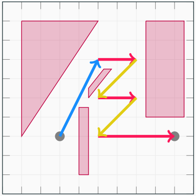
PTIME-complete* (Alur et al. HSCC'12)		Decidable (Krishna et al. ATVA'17)

* Safe reachability with Zone $= \mathbb{R}^d_{\geq 0}$ is also PTIME-complete by work on continuous VAS / Petri nets (Fraca and Haddad PN'13)

Our questions

- Can we unify these results?
- What are the decidable problems?
- What is their complexity?

Linear temporal logic (LTL): syntax

$$\varphi ::= \mathit{true} \mid Z \mid \neg\varphi \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid \mathsf{F}\varphi \mid \mathsf{G}\varphi \mid \varphi \,\mathsf{U}\, \varphi$$

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Zones: closed convex polytopes

Linear temporal logic (LTL): syntax

$$\varphi ::= \mathit{true} \mid Z \mid \neg\varphi \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid \mathsf{F}\varphi \mid \mathsf{G}\varphi \mid \varphi \;\mathsf{U}\; \varphi$$

Finally φ holds

Linear temporal logic (LTL): syntax

$$\varphi ::= \mathit{true} \mid Z \mid \neg\varphi \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid \mathsf{F}\varphi \mid \mathsf{G}\varphi \mid \varphi\ \mathsf{U}\ \varphi$$

Globally φ holds

Linear temporal logic (LTL): syntax

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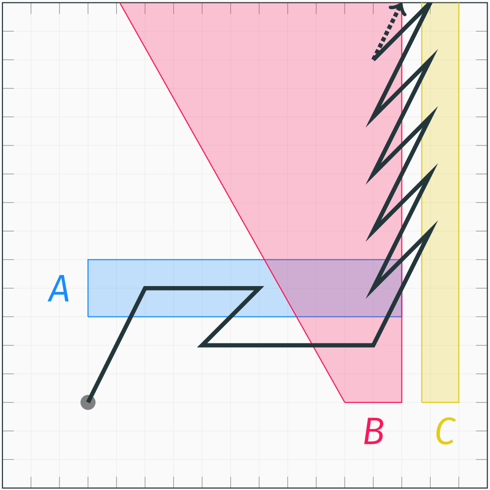
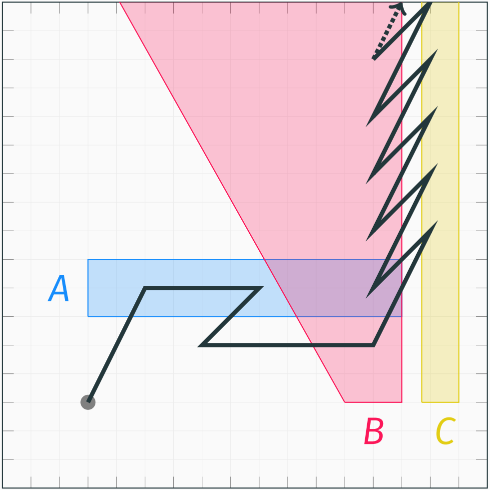
φ holds until φ' holds

Linear temporal logic (LTL): informal semantics



Linear temporal logic (LTL): informal semantics

$\sigma \models \mathsf{F}\, A$



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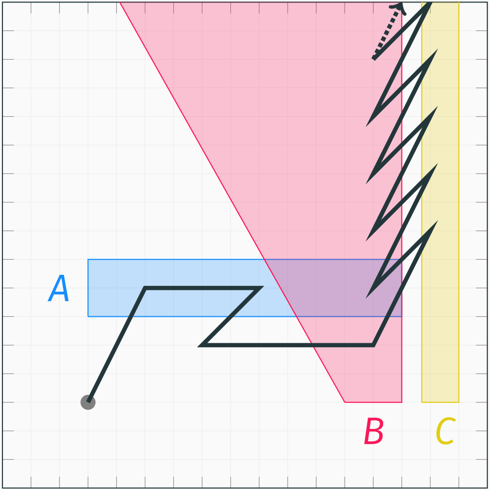

$\sigma \models \mathsf{F}\, A$

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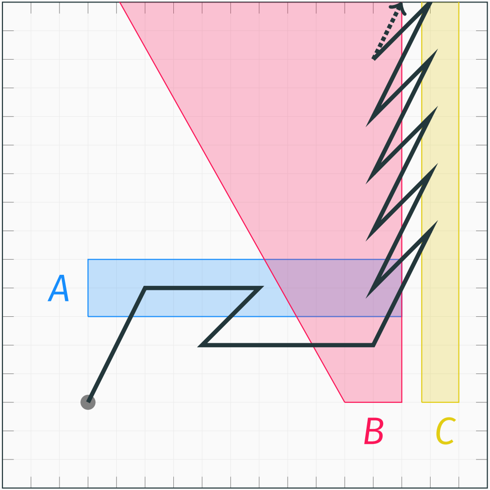
$\sigma \models \mathsf{F}\,(A \wedge \mathsf{F}\, B)$



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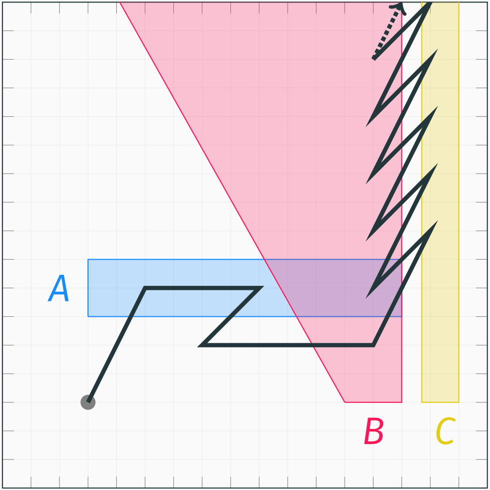

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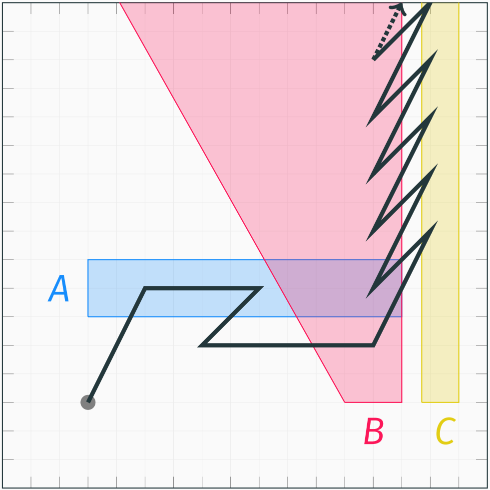
$\sigma \models (\neg C)\ \mathsf{U}\ B$



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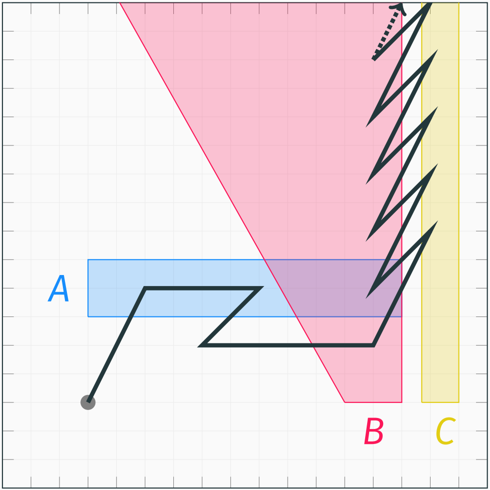

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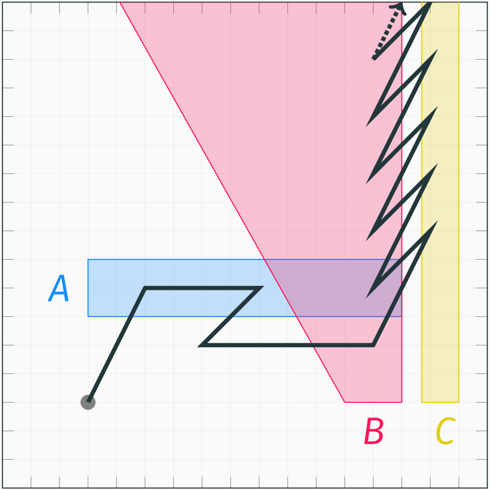
$\sigma \models (\mathsf{GF}\, B) \wedge (\mathsf{GF}\, C)$



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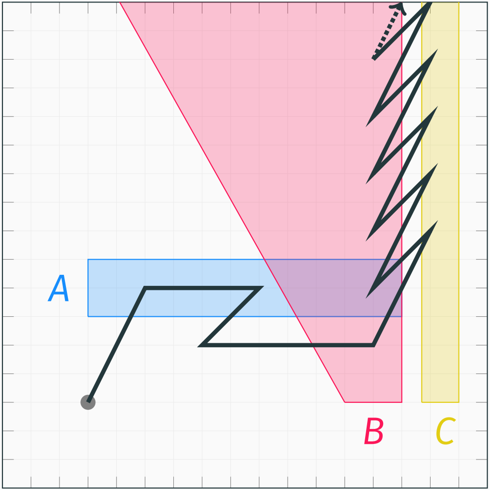
$\sigma \models (\mathsf{GF}\, B) \wedge (\mathsf{GF}\, C)$

$\sigma \not\models \mathsf{GF}(B \wedge C)$



Linear temporal logic (LTL): formal semantics

$$\sigma, \tau \models \mathit{true} \iff \mathit{true}$$

$$\sigma, \tau \models Z \iff \sigma(\tau) \in Z$$

Time $\tau \in \mathbb{R}_{\geq 0}$

Execution $\sigma : \mathbb{R}_{\geq 0} \to \mathbb{R}^d$

Linear temporal logic (LTL): formal semantics

$$\sigma, \tau \models \mathit{true} \iff \mathit{true}$$

$$\sigma, \tau \models Z \iff \sigma(\tau) \in Z$$

$$\sigma, \tau \models \neg\varphi \iff \neg(\sigma, \tau \models \varphi)$$

Linear temporal logic (LTL): formal semantics

$$\begin{aligned}
\sigma, \tau \models \mathit{true} &\iff \mathit{true} \\
\sigma, \tau \models Z &\iff \sigma(\tau) \in Z \\
\sigma, \tau \models \neg\varphi &\iff \neg(\sigma, \tau \models \varphi) \\
\sigma, \tau \models \varphi \wedge \varphi' &\iff (\sigma, \tau \models \varphi) \wedge (\sigma, \tau \models \varphi')
\end{aligned}$$

Linear temporal logic (LTL): formal semantics

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\sigma, \tau \models \varphi \vee \varphi' &\iff (\sigma, \tau \models \varphi) \vee (\sigma, \tau \models \varphi')
\end{aligned}
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Linear temporal logic (LTL): formal semantics

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\sigma, \tau \models \mathsf{F}\varphi &\iff \exists \tau' \geq \tau : \sigma, \tau' \models \varphi
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\sigma, \tau \models \mathsf{G}\varphi &\iff \forall \tau' \geq \tau : \sigma, \tau' \models \varphi
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Linear temporal logic (LTL): formal semantics

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\sigma, \tau \models \mathsf{G}\varphi &\iff \forall \tau' \geq \tau : \sigma, \tau' \models \varphi \\
\sigma, \tau \models \varphi \,\mathsf{U}\, \varphi' &\iff \exists \tau' \geq \tau : (\sigma, \tau' \models \varphi') \wedge (\forall \tau'' \in [\tau, \tau') : \sigma, \tau'' \models \varphi)
\end{aligned}$$

Linear temporal logic (LTL): formal semantics

$$\sigma \models \varphi \iff \sigma, 0 \models \varphi$$

Model checking: problem

$\boldsymbol{x} \models_M \varphi$ iff there is a schedule π of M such that $\mathrm{exec}(\pi, \boldsymbol{x}) \models \varphi$

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Model checking problem

Given: *MMS M, initial point $\boldsymbol{x}$, LTL formula φ*

Decide: *whether $\boldsymbol{x} \models_M \varphi$*

Model checking: problem

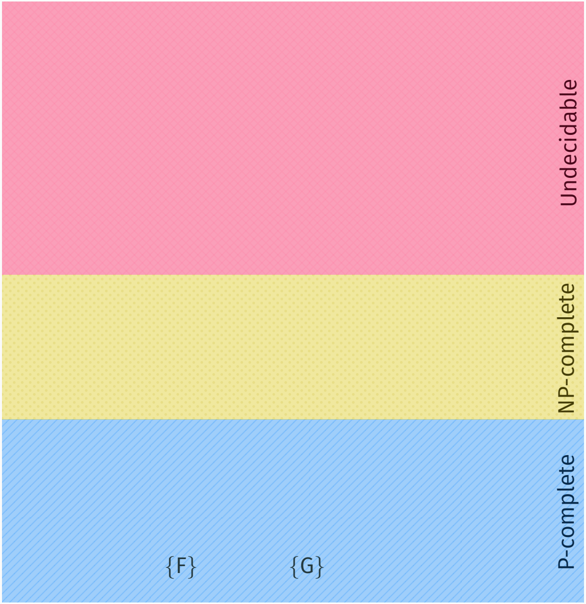
$\boldsymbol{x} \models_M \varphi$ iff there is a schedule π of M such that $\mathrm{exec}(\pi, \boldsymbol{x}) \models \varphi$

Model checking problem$_{X \subseteq \{F, G, U, \wedge, \vee, \neg\}}$

Given: *MMS M, initial point $\boldsymbol{x}$,*
LTL formula φ with operators only from X

Decide: *whether $\boldsymbol{x} \models_M \varphi$*

Model checking: our contribution



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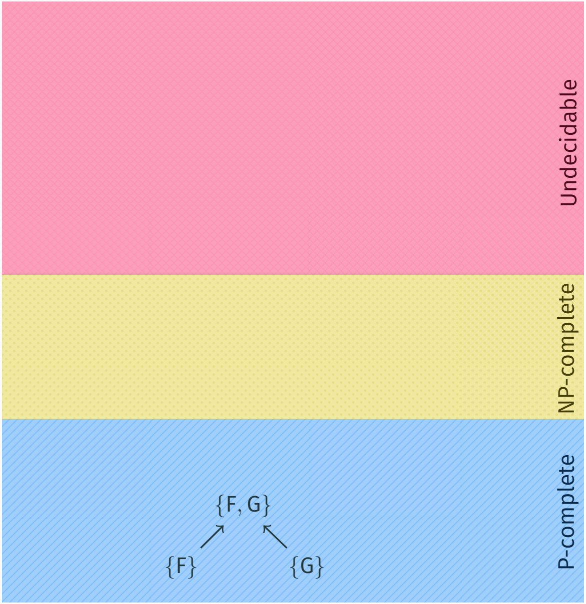

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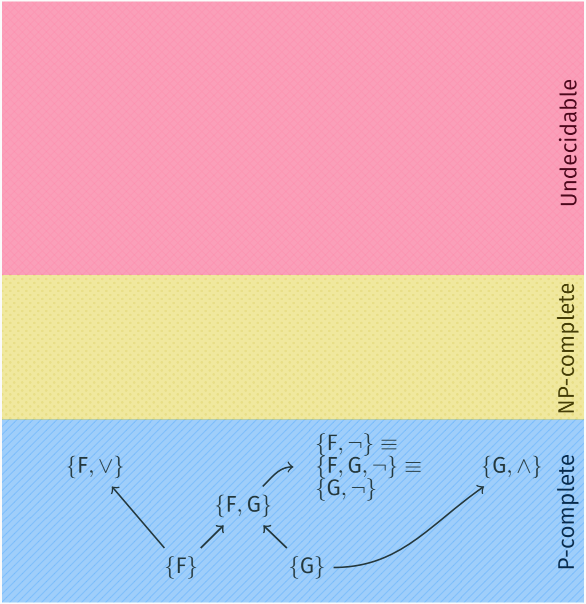

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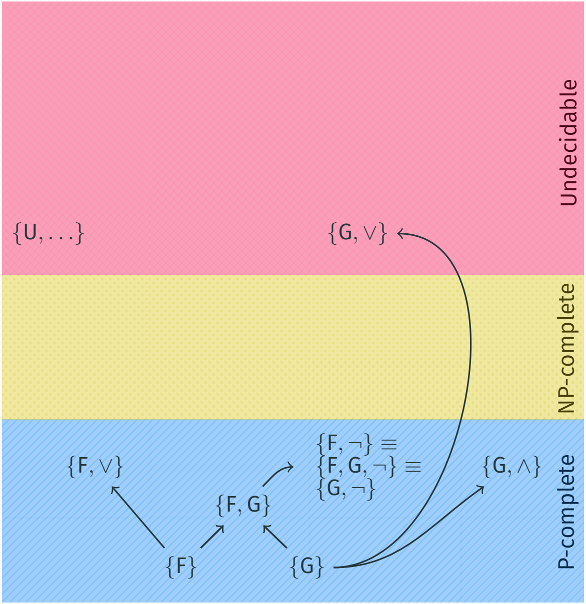

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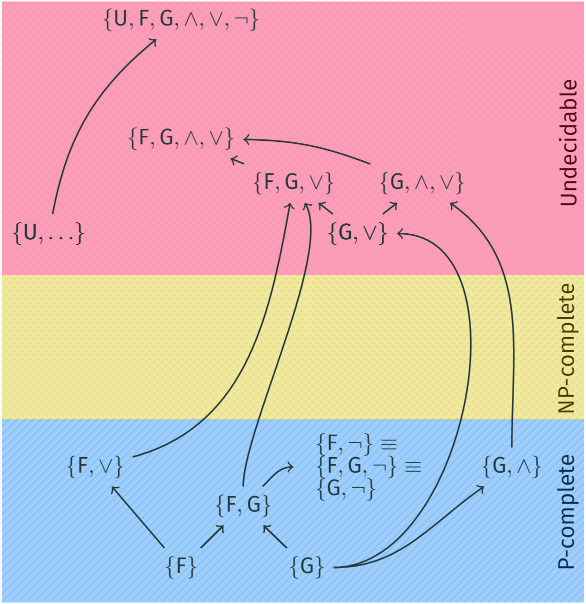

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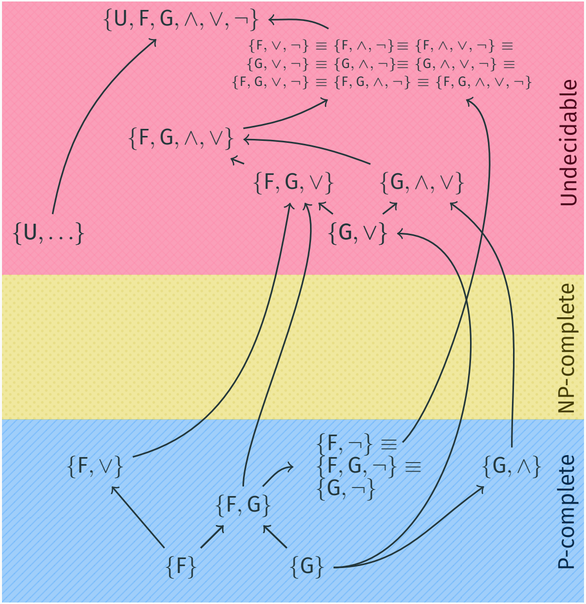

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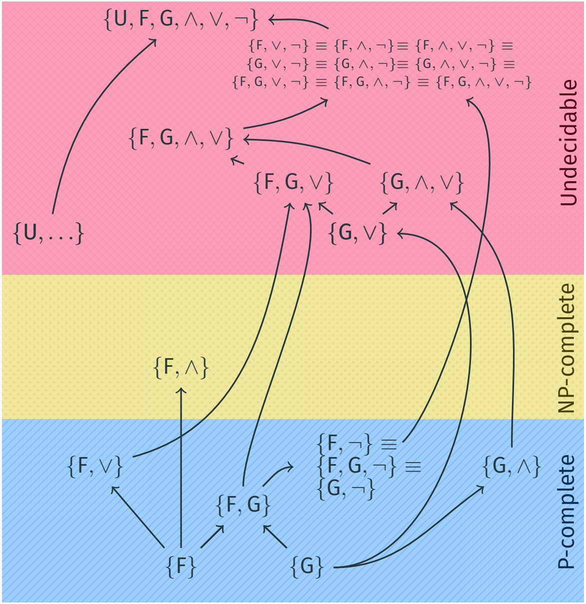

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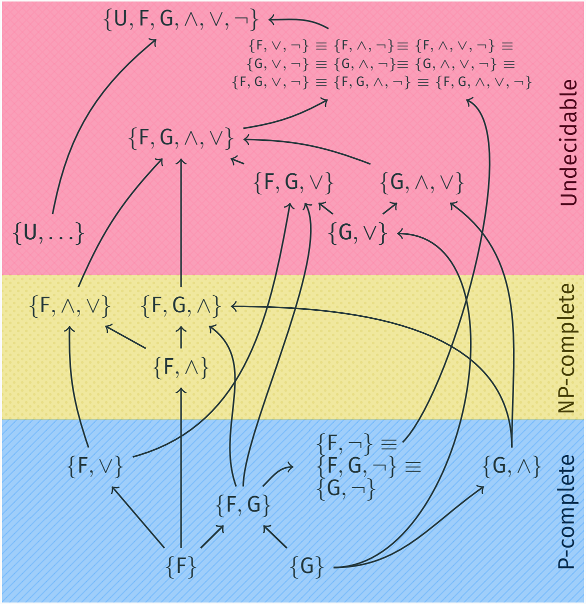

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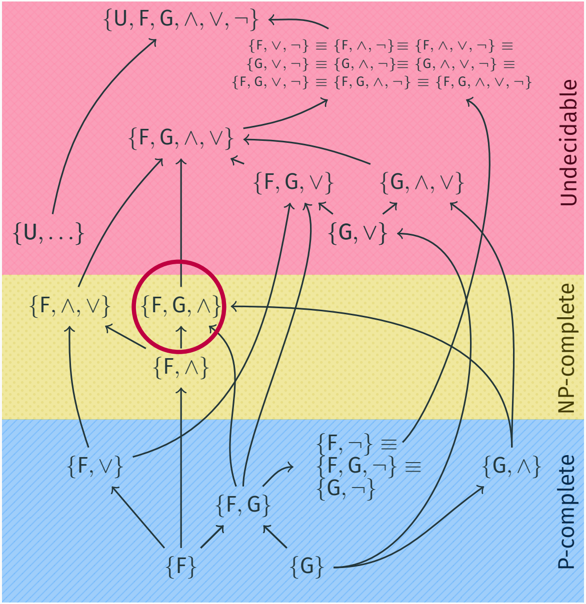

$\{\mathbf{F}, \mathbf{G}, \land\} \in$ NP: overview

1. Flatten formula φ

$$\varphi \xrightarrow{①} \text{flat}(\varphi)$$

$\{\mathbf{F}, \mathbf{G}, \wedge\} \in$ NP: overview

1. Flatten formula φ
2. Convert into an almost acyclic automaton $\mathcal{A}_\varphi$

$$\varphi \xrightarrow{①} \text{flat}(\varphi) \xrightarrow{②} \mathcal{A}_\varphi$$

$\{\mathbf{F}, \mathbf{G}, \wedge\} \in$ NP: overview

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① Formula flattening

Definition

An LTL formula is *flat* if it can be derived from φ in

$$\varphi ::= \mathrm{goal} \mid \mathsf{G}\,\mathrm{goal} \mid \mathsf{GF}\,\mathrm{goal} \mid \mathsf{F}\,\varphi \mid \varphi \wedge \varphi$$
$$\mathrm{goal} ::= \mathit{true} \mid Z \mid \mathrm{goal} \wedge \mathrm{goal}$$

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Intuition: only F
can nest complex goals

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Example

Formula	Equivalent flat formula
$\mathsf{GF}(A \wedge \mathsf{G}\,B \wedge \mathsf{F}\,C)$	$\mathsf{GF}\,A \wedge \mathsf{FG}\,B \wedge \mathsf{GF}\,C$

① Formula flattening

Theorem

For every $\varphi \in$ *LTL*$(F, G, \wedge)$
There is an equivalent flat formula $\mathrm{flat}(\varphi)$ *of linear size*

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Proof.

Follows by simple recursive rewriting rules □

② From flat formulas to almost acyclic automata

$$\mathsf{GF}\,A \wedge \mathsf{FG}\,C \wedge \mathsf{F}\,B$$

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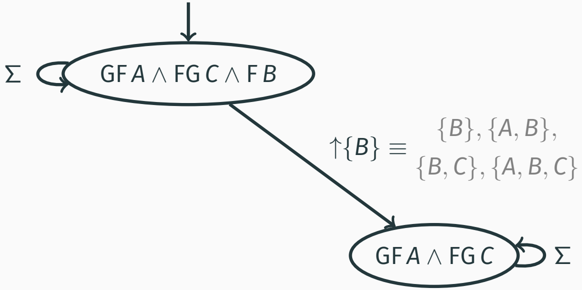
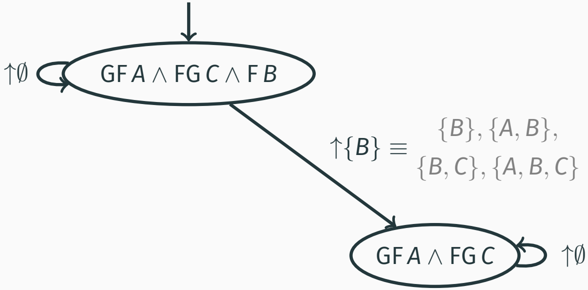


② From flat formulas to almost acyclic automata



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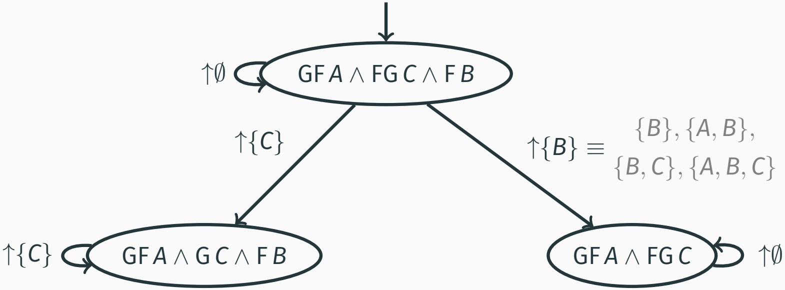

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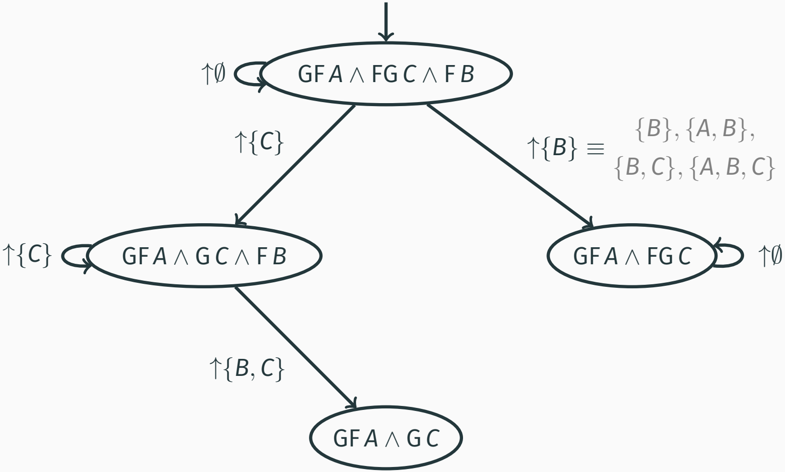

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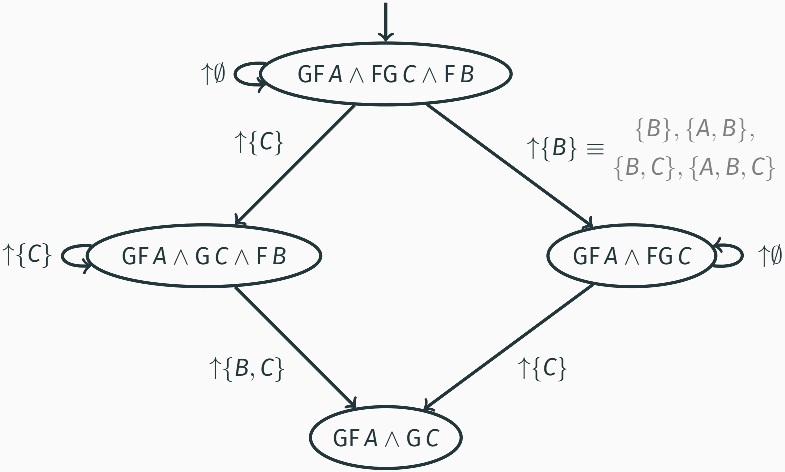

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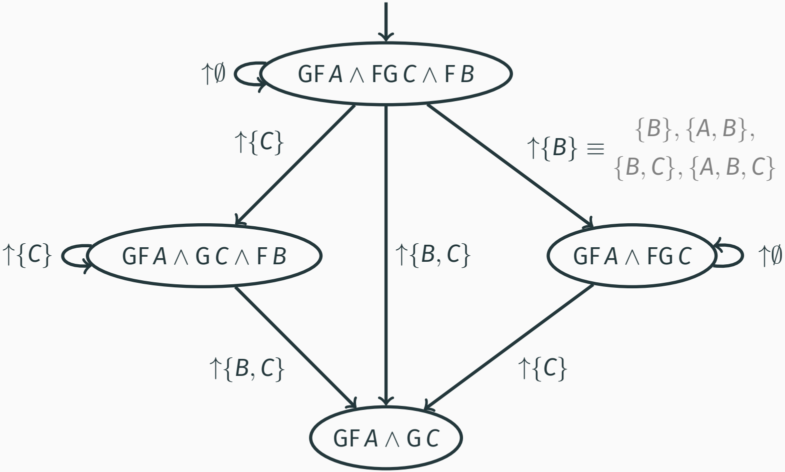

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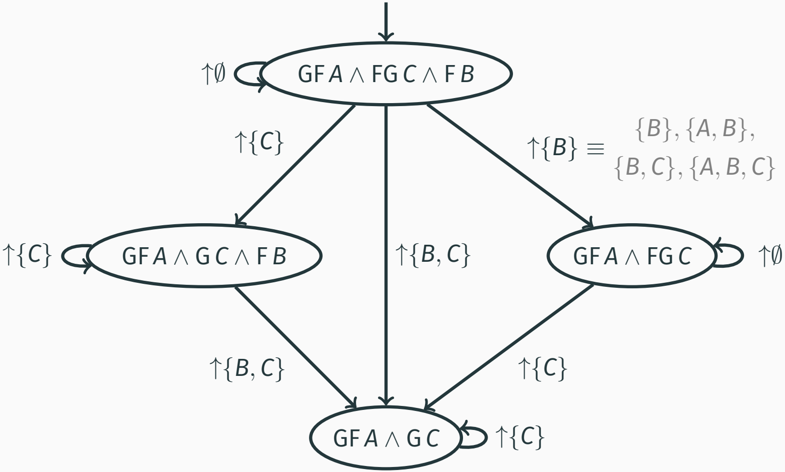

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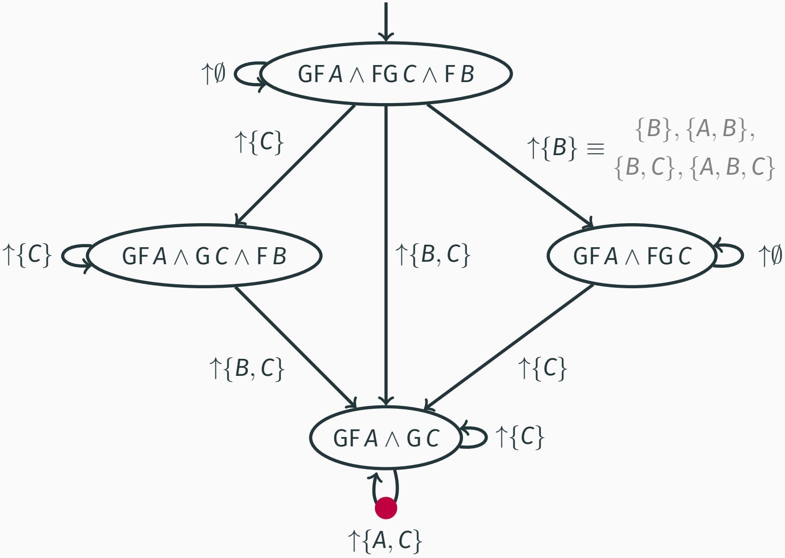

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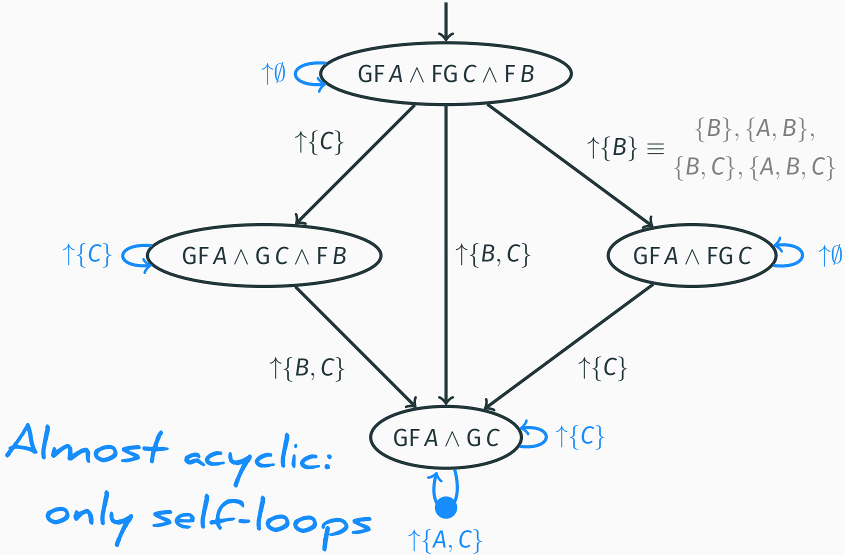

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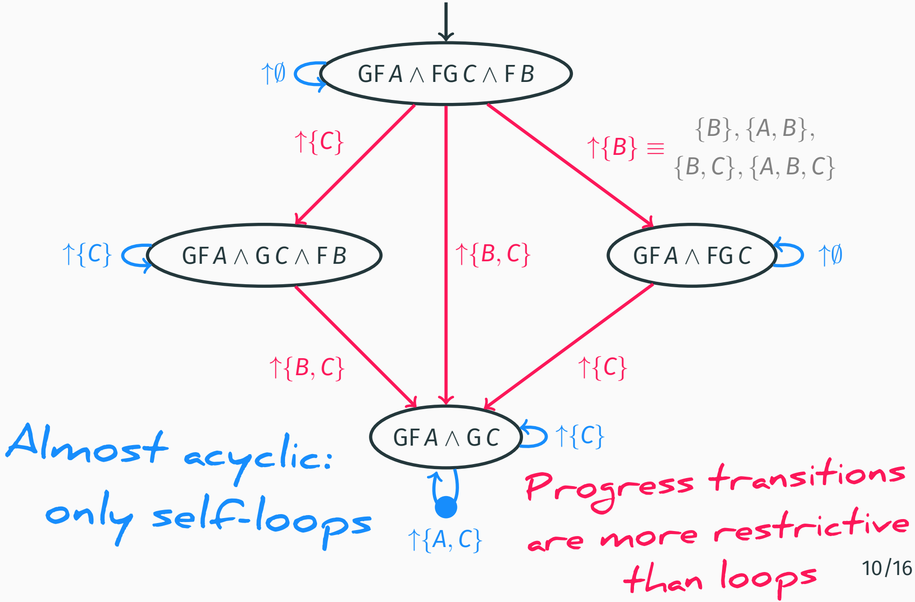

② From flat formulas to almost acyclic automata

Theorem

For every formula $\varphi \in LTL(F, G, \wedge)$
There is an almost acylic automaton $\mathcal{A}_\varphi$ s.t.

- $\text{language}(\mathcal{A}_\varphi) \equiv \varphi$
- $\text{width}(\mathcal{A}_\varphi) \in \mathcal{O}(|\varphi|)$
- *transitions have "good properties"*

Proof.

Inspired by unfoldings of Křetínský and Esparza CAV'12 □

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Theorem

For every formula $\varphi \in LTL(F, G, \wedge)$
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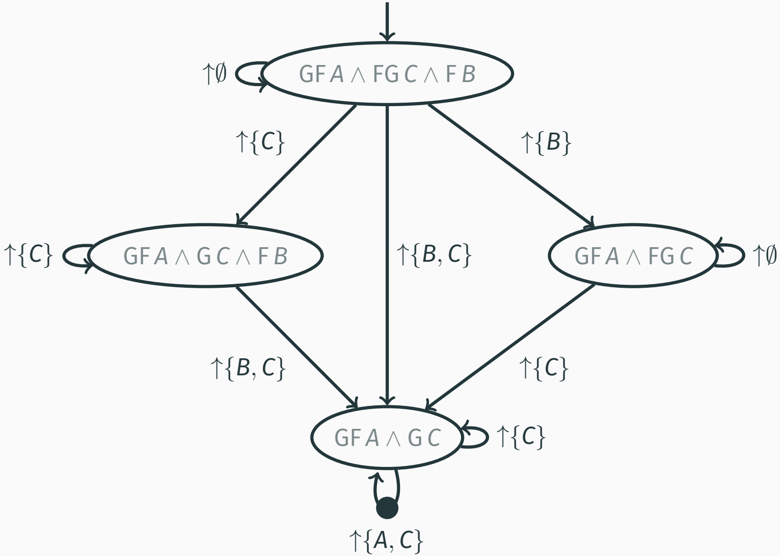
- *transitions have "good properties"*

(Generalized Büchi automaton with transition-acceptance)

Proof.

Inspired by unfoldings of Křetínský and Esparza CAV'12 □

③ Linear path schemes (LPS)



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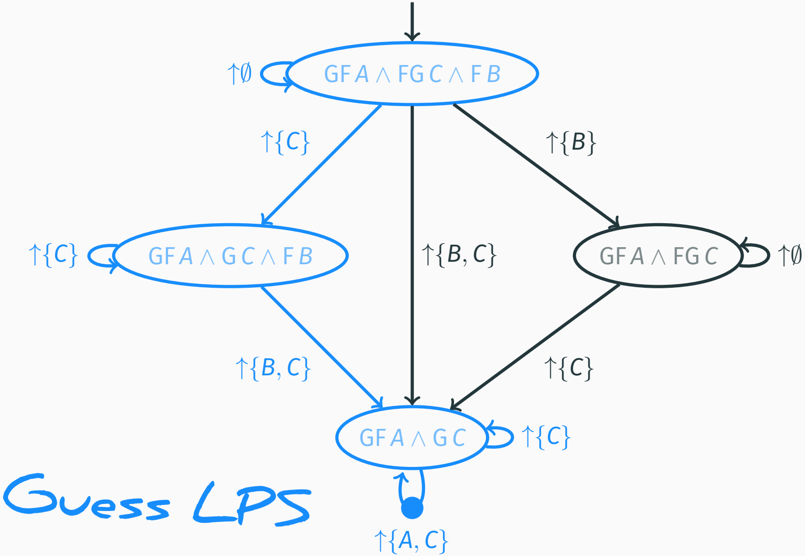
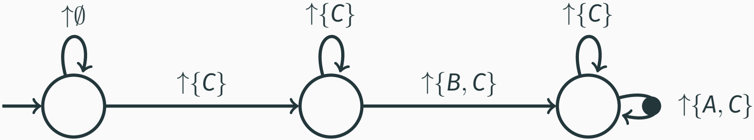


④ From LPS back to linear LTL



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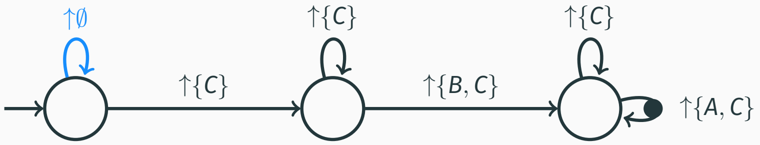
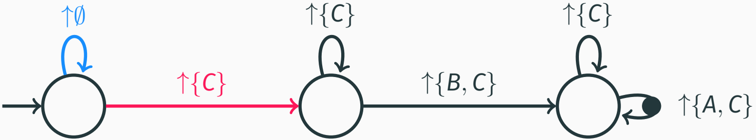


$$\mathit{true}\ \mathsf{U}\ (C \wedge (C\ \mathsf{U}\ ((B \wedge C) \wedge (\mathsf{G}\mathsf{F}\, A \wedge \mathsf{G}\, C))))$$



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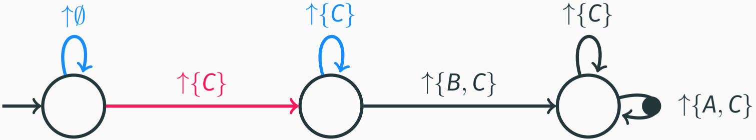

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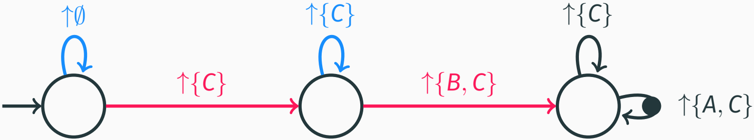

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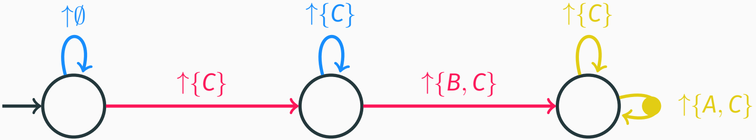

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④ From LPS back to linear LTL

$$\boldsymbol{x} \models_M \mathit{true} \; \mathrm{U} \; (C \wedge (C \; \mathrm{U} \; ((B \wedge C) \wedge (\mathrm{GF}\, A \wedge \mathrm{G}\, C))))$$

④ From LPS back to linear LTL

$$\boldsymbol{x} \models_M \mathit{true}\ \mathrm{U}\ (C \wedge (C\ \mathrm{U}\ ((B \wedge C) \wedge (\mathrm{GF}\, A \wedge \mathrm{G}\, C))))$$

iff

$$\exists \boldsymbol{y} \in C,\ \boldsymbol{z} \in B \cap C : \boldsymbol{x} \rightarrow^* \boldsymbol{y} \rightarrow^*_C \boldsymbol{z} \text{ and } \boldsymbol{z} \models_M \mathrm{GF}\, A \wedge \mathrm{G}\, C$$

Convex semi-linear Horn formulas

Such a formula can be checked in polynomial time:

$$\exists \boldsymbol{x} \in \mathbb{R}^d : \bigwedge_i \boldsymbol{a}_i \boldsymbol{x} \sim_i b_i$$

with $\sim_i \in \{<, \leq, =, \geq, >\}$

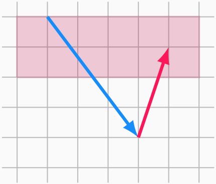
Convex semi-linear Horn formulas **(B. and Haase LICS'17)**

Such a formula can be checked in polynomial time:

$$\exists \boldsymbol{x} \in \mathbb{R}^d, \boldsymbol{x}' \in \mathbb{R}^{d'}_{\geq 0} : \bigwedge_i \left(\boldsymbol{a}_i \boldsymbol{x} + \boldsymbol{a}'_i \boldsymbol{x}' \sim_i b_i \vee \bigvee_j \bigwedge_k \boldsymbol{x}'(k) > 0 \right)$$

with $\sim_i \in \{<, \leq, =, \geq, >\}$

⑤ From linear LTL to FO: safe reachability



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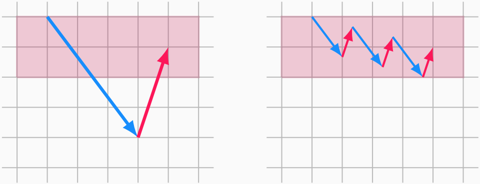

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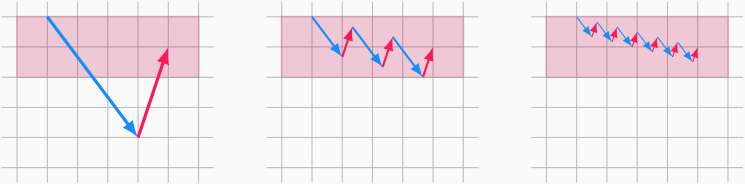

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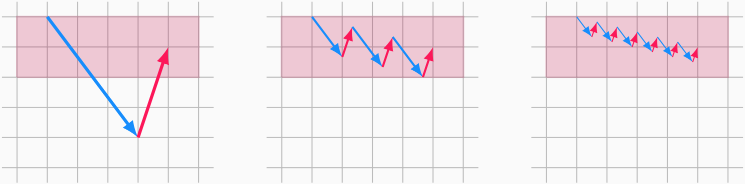


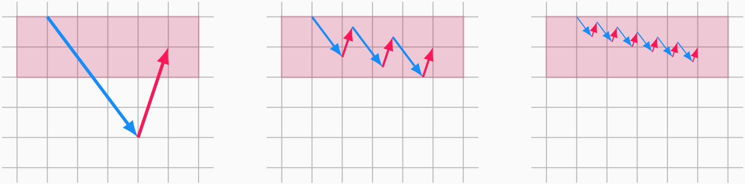
Proposition (generalization from Fraca and Haddad PN'13)

We have $\boldsymbol{x} \rightarrow_Z^* \boldsymbol{y}$ iff there exist schedules $\pi, \pi_{\text{fwd}}, \pi_{\text{bwd}}$ with

(i) $\boldsymbol{x} \rightarrow^{\pi} \boldsymbol{y}$ (ii) $\boldsymbol{x} \rightarrow_Z^{\pi_{\text{fwd}}} \cdot$ (iii) $\cdot \rightarrow_Z^{\pi_{\text{bwd}}} \boldsymbol{y}$

and $\text{supp}(\pi) = \text{supp}(\pi_{\text{fwd}}) = \text{supp}(\pi_{\text{bwd}})$

⑤ From linear LTL to FO: safe reachability



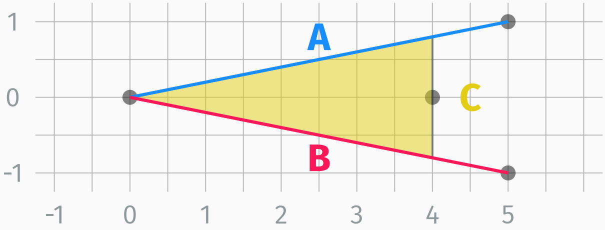
Proposition **(generalization from B. and Haase LICS'17)**

There is a convex semi-linear Horn formulas ψ_Z s.t.

$$\psi_Z(\boldsymbol{x}, \boldsymbol{y}) \iff \boldsymbol{x} \to_Z^* \boldsymbol{y}$$

⑤ From linear LTL to FO: safe repeated reachability

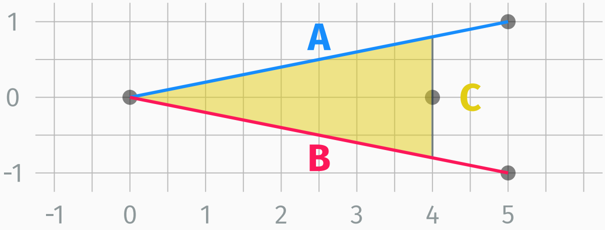
$$\mathsf{GF}\,A \wedge \mathsf{GF}\,B \wedge \mathsf{G}\,C$$



$$\boldsymbol{m}_1 = (0, 0, 1) \qquad \boldsymbol{m}_2 = (0, 0, -1)$$

$$\boldsymbol{m}_3 = (-1, 2, 0) \qquad \boldsymbol{m}_4 = (-1, -2, 0)$$

$$\mathrm{GF}\,A \wedge \mathrm{GF}\,B \wedge \mathrm{G}\,C$$



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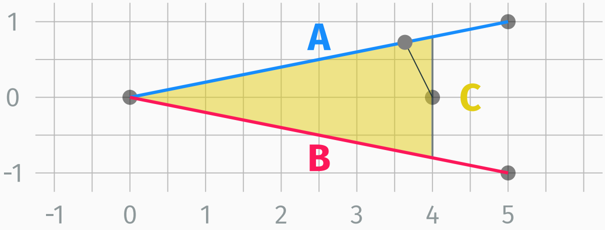
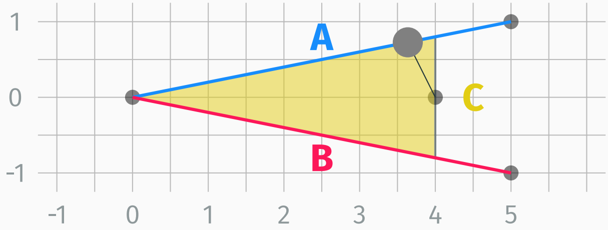


$\frac{4}{11}\boldsymbol{m}_3$

⑤ From linear LTL to FO: safe repeated reachability

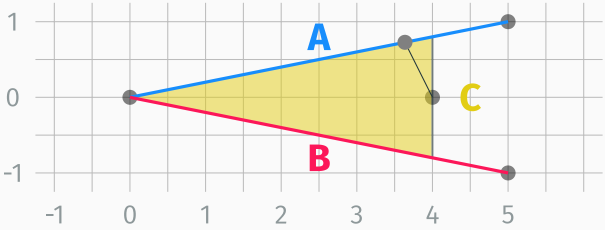
$$\mathsf{GF}\,A \wedge \mathsf{GF}\,B \wedge \mathsf{G}\,C$$



$\frac{4}{11}\boldsymbol{m}_3\ \boldsymbol{m}_1$

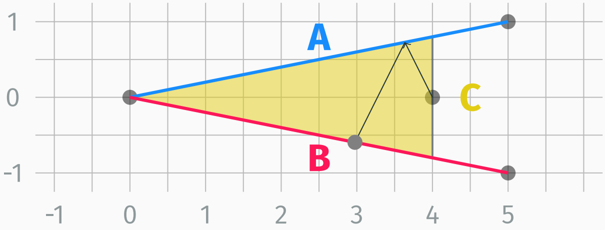
⑤ From linear LTL to FO: safe repeated reachability

$$\mathsf{GF}\,A \wedge \mathsf{GF}\,B \wedge \mathsf{G}\,C$$



$\frac{4}{11}\boldsymbol{m}_3\,\boldsymbol{m}_1\,\boldsymbol{m}_2$

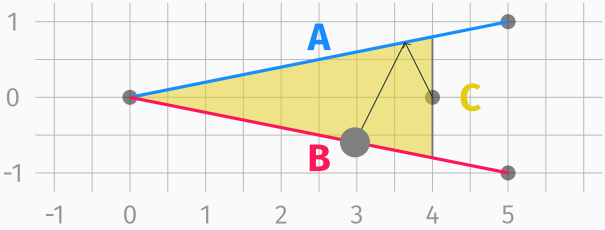
$$\mathsf{GF}\, A \wedge \mathsf{GF}\, B \wedge \mathsf{G}\, C$$



$$\frac{4}{11}\boldsymbol{m}_3\ \boldsymbol{m}_1\ \boldsymbol{m}_2\ \frac{80}{121}\boldsymbol{m}_4$$

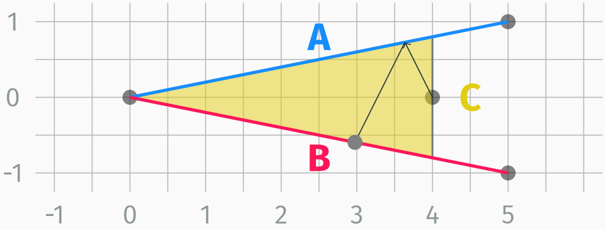
⑤ From linear LTL to FO: safe repeated reachability

$$\mathsf{GF}\,A \wedge \mathsf{GF}\,B \wedge \mathsf{G}\,C$$



$$\tfrac{4}{11}\boldsymbol{m}_3\ \boldsymbol{m}_1\ \boldsymbol{m}_2\ \tfrac{80}{121}\boldsymbol{m}_4\ \boldsymbol{m}_1$$

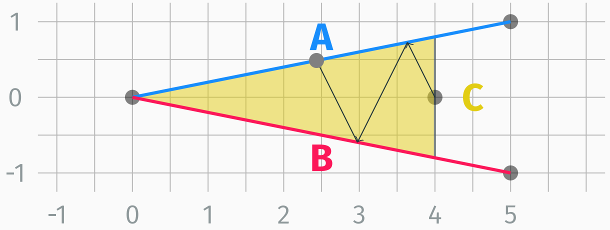
$$\mathrm{GF}\,A \wedge \mathrm{GF}\,B \wedge \mathrm{G}\,C$$



$$\tfrac{4}{11}\boldsymbol{m}_3\, \boldsymbol{m}_1\, \boldsymbol{m}_2\, \tfrac{80}{121}\boldsymbol{m}_4\, \boldsymbol{m}_1\, \boldsymbol{m}_2$$

⑤ From linear LTL to FO: safe repeated reachability

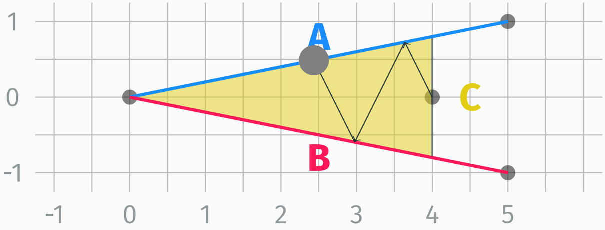
$$\mathsf{GF}\,A \wedge \mathsf{GF}\,B \wedge \mathsf{G}\,C$$



$$\tfrac{4}{11}\boldsymbol{m}_3\,\boldsymbol{m}_1\,\boldsymbol{m}_2\,\tfrac{80}{121}\boldsymbol{m}_4\,\boldsymbol{m}_1\,\boldsymbol{m}_2\,\tfrac{720}{1331}\boldsymbol{m}_3$$

⑤ From linear LTL to FO: safe repeated reachability

$$\mathrm{GF}\,A \wedge \mathrm{GF}\,B \wedge \mathrm{G}\,C$$



$$\tfrac{4}{11}\boldsymbol{m}_3\,\boldsymbol{m}_1\,\boldsymbol{m}_2\,\tfrac{80}{121}\boldsymbol{m}_4\,\boldsymbol{m}_1\,\boldsymbol{m}_2\,\tfrac{720}{1331}\boldsymbol{m}_3\,\boldsymbol{m}_1$$

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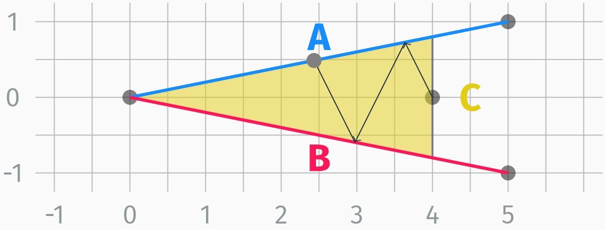

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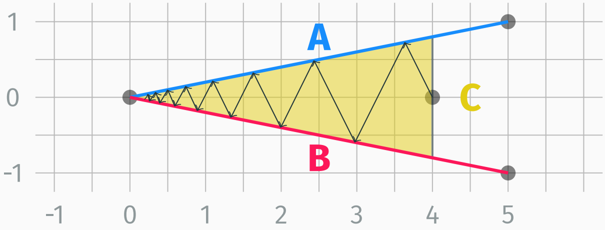

$$\tfrac{4}{11}\boldsymbol{m}_3\,\boldsymbol{m}_1\,\boldsymbol{m}_2\,\tfrac{80}{121}\boldsymbol{m}_4\,\boldsymbol{m}_1\,\boldsymbol{m}_2\,\tfrac{720}{1331}\boldsymbol{m}_3\,\boldsymbol{m}_1\,\boldsymbol{m}_2\,\cdots$$

⑤ From linear LTL to FO: safe repeated reachability

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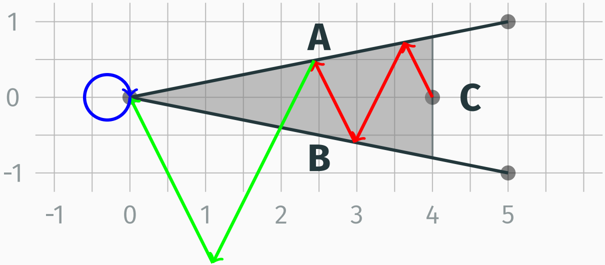

1. Safe exec. (SE) 2. Inter. exec. (IE) 3. Loop (L)

⑤ From linear LTL to FO: safe repeated reachability

Informal theorem

There is a safe exec. (SE) from $\boldsymbol{x}$, an intermediate exec. (IE) and a loop (L) s.t. $\emptyset \neq \text{supp}(L) \subseteq \text{supp}(SE) = \text{supp}(IE)$

iff

$$\boldsymbol{x} \models_M \mathsf{GF}\, A \wedge \mathsf{GF}\, B \wedge \mathsf{G}\, C$$

Proof sketch of $\Leftarrow$

(SE) follows by definition

(IE, L) follows from Farkas' lemma

⑤ From linear LTL to FO: safe repeated reachability

Informal theorem

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Expressible as a
convex semi-linear Horn formula

$\{\mathbf{F}, \mathbf{G}, \wedge\} \in$ NP: recap

1. Flatten formula φ
2. Convert into an almost acyclic automaton $\mathcal{A}_\varphi$
3. Nondeterministically guess a “path” π of $\mathcal{A}_\varphi$
4. Convert π into a “linear” LTL formula φ'
5. Construct a first-order formula ψ s.t. $\psi(\boldsymbol{x}) \leftrightarrow \boldsymbol{x} \models_M \varphi'$
6. Check whether $\psi(\boldsymbol{x})$ holds (in polynomial time)

$$\varphi \xrightarrow{①} \mathrm{flat}(\varphi) \xrightarrow{②} \mathcal{A}_\varphi \xrightarrow{③} \pi \xrightarrow{④} \varphi' \xrightarrow{⑤} \psi \; ⑥ \begin{cases} \nearrow \text{Yes} \\ \searrow \text{No} \end{cases}$$

Conclusion: summary

- Introduced LTL for MMS
- Classified each syntactic fragment as P-c., NP-c. or undecidable
- Generalizes and unifies results on continuous counter systems

Conclusion: future work

- Handling richer properties
- Practical implementation
- Extension of LTL to 2-player games

Thank you!
Vielen Dank!